

# **A Stroll Through Space-Time**

*A Leisurely Discourse on  
Einstein's Relativity Theory*

*Rakesh Popli*



*Vigyan Prasara*

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**Dedicated  
to  
the Inspiring Memory of  
DR. RAJENDRA KUMAR  
(University of Delhi)**

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# Foreword

Vigyan Prasara brought out a number of publications on a variety of topics of science and technology on an experimental basis. Popular Science classics, India's Scientific Heritage, Natural History, Health, and Do-It-Yourself are some of the series that have evolved over the years. Our emphasis has been on bringing out quality publications on various aspects of science and technology at affordable prices. Further, Vigyan Prasara is putting in efforts to bring out publications in major Indian languages for various target groups.

The present book by Professor Rakesh Popli is an attempt to present a popular account of Einstein's theory of relativity. It is intended that even people without any mathematical background would be able to savour the basic ideas and consequences of the theory of the relativity. At the same time students of Physics also would find it interesting.

New Delhi  
17 November 2003

**Vinay. B. Kamble**  
Acting Director  
Vigyan Prasara

## Preface

Einstein's Theory of Relativity has a unique charm, not only for the specialist scientist, but also for the layman. The strange and mysterious ways, in which lengths contract, time loses its absolute character and masses of gas collapse into black holes, fascinate and puzzle the human mind to no end. And that famous and esoteric equation,  $E = mc^2$ , which expresses the equivalence of mass and energy, gives rise to wonder and many questions.

For all the curiosity and fascination it arouses, the Theory of Relativity is even to this day shut up in a remote cave in a deep jungle, so to say. Laymen dare not tread the tortuous path leading to it. Even students of physics often get so badly bruised and weary in stumbling through the rocky footpaths and thorny bushes of complex equations, tensors, four-dimensional space-time representations and transformations between reference-frames that they fail to enjoy the beauty and harmony of the Theory. There is need for popular literature on this subject that by-passes mathematical formalism for the most part and allows the reader to get acquainted directly with the concept and its implications.

At the international level, a good deal of such popular literature has been produced by top scientists and celebrated writers. However, for a theory as fascinating and intriguing as this, there is always scope for different presentations. Moreover, there is hardly any popular literature written by *Indian authors*, in the *Indian context* and in *Indian languages*. The present work, being produced in Hindi as well as in English, is a humble attempt to fill this gap.

The Theory of Relativity (General as well as Special) brings home the realization that 'self-evident' statements about space and time need not necessarily be true. For example, we often take it as 'self-evident' that "time flows at the same rate for all observers." However, it is possible that, outside the limits of everyday experiences, this 'self-evident' truth contradicts some other fundamental law of nature. It does not help to evade such a contradiction by mixing it with metaphysics or covering it with a shroud of mystery. A new theory has to be created which re-establishes the harmony of the laws of nature, and also agrees with experimental results. Not only that, the new theory may predict the results of unobserved processes, which can be verified by further experiments. Such a theory becomes a milestone in the development of science as Einstein's Relativity Theory has.

This book is addressed particularly to the educated layman. There is every hope, however, that students of science including physics will also find it interesting and useful in clarifying their concepts and answering their questions (and raising new ones).

The book starts with a biographical note on Albert Einstein, to give the reader a glimpse into the personality and way of thinking of the wonderful creator of this wonderful theory. The first chapter is devoted to explaining the meanings of such concepts as absolute and relative, relativity of directions, and reference-frames. The second chapter deals with concepts of space and time in the Newtonian and Einsteinian schemes, and attempts to show clearly to the reader how space and time get inter-twined in Einstein's scheme and what role light plays in connecting the two. This paves the way for discussing some of the characteristic results of the Special Theory of Relativity such as length contraction and time dilation in Chapter Three. The next chapter discusses in some detail the equivalence of Mass and Energy and its significance, particularly in dealing with atomic energy.

Readers, including students of science, who wonder what kind of creature the four-dimensional space-time is, will find a discussion of this concept in Chapter Five. A preliminary discussion of the General Theory of Relativity and its consequences has been included in Chapter Six because the book would have been incomplete without it. It is hoped that even laymen will be able to follow the treatment of these two abstract (but most interesting) topics. The last chapter presents some hilarious and mind-boggling paradoxes based on the Theory of Relativity.

In preparing this book, I have freely taken advantage of the insight and ways of explanation contained in several books. I am grateful to Dr. Narender Sehgal, Dr. S. Mollah, Dr. Gagan Gupta, Siddhartha Sengupta and Smt. Rama Popli for going through parts of the manuscript and offering useful suggestions. I offer thanks to Kaushik Pal and Amitabh Mukherjee for drawing the beautiful illustrations and M.M. Mishra for typing the manuscript. The Vigyan Prasara staff are to be thanked for their friendly co-operation and efficient processing of the manuscript.

# Albert Einstein & Pursuit of Scientific Harmony

Writing about Einstein's life, that too in a few pages, is not easy. How could a boy, who was never considered bright by elders at home or school, who quit the gymnasium (high school) before completing his studies (or was eased out?), become a super-giant who shook the entire world of science? How could his name become synonymous with twentieth-century science? It is a most wonderful story. No less intriguing is the question how, having reached the pinnacle of brilliance and fame, he became so isolated among mainstream scientists.



Not only did Einstein bring about a revolution in science, he also challenged the frenzy of racism and war, and exposed the hollowness of the glitter of worldly possessions. Einstein's life-story, if read carefully and sincerely, can even bring about an upheaval in the education-system of a country like India.

Albert Einstein was born in 1879 in Ulm, a small town of Swabia (South-West Germany). His father ran a small business that never yielded much profit. Nothing remarkable is heard about Albert's childhood, except that he was very slow in learning to speak (retarded, perhaps?), and when troops marched the streets to the tune of the military band, he would not get excited like other children but would cling to his father.

Einstein witnessed from his childhood the struggle between the social-cultural milieu of Swabia and that of the Prussian State. Swabia had deep-rooted rationalist traditions, which were sought to be replaced with a blind faith in the infallibility of the police state by officers of the Prussian Empire.



Even the melodious speech of Swabians contrasted with the harsh, chopped speech of Prussians. Albert chose reason, justice and humanism from the very beginning—so much so that he became an embodiment of these.

Albert's school-life was not to his liking. Teachers would have the students cram Greek and Latin grammar, tedious chronologies of history, facts of science, etc. in a spirit similar to that of military officers. (If only the medium of instruction had been a remote foreign language, the situation would have become similar to 'superior' public schools of India!). Albert's slow manner of speech would exasperate the teachers. They had neither the patience nor the capacity to fathom the depth and precision of his answers. He started taking lessons in violin at the age of six. There, too, the school-like stereotyped lessons failed to inspire him. It took Mozart's sonatas to awaken him to the emotions and beauty of music. He practiced music with all his might so that he might play Mozart's compositions.

It was Albert's uncle, an engineer, who instilled in him an interest in mathematics. Albert read most lessons of maths before these were taught in the school. Meanwhile, an acquaintance introduced him to the series *Popular Books on Natural Science* compiled by Aaron Bernstein (in German). These and other books brought home to him the inter-dependence of all zoological, botanical, geological, astronomical and other facets of nature and the universal principles underlying their inter-relationships. These books greatly reinforced the ideas of reason and inner harmony rooted deeply in his mental make-up.

On the other hand, the school taught the origin of the world and life on the basis of the Biblical religion, which was contrary to facts mentioned in the science books. This dichotomy made a strong impression on him. He developed an aversion for dogmatic religions of all kinds and devoted himself to science and harmony.

Upon leaving the gymnasium in Munich, Albert toured around for a while in Italy, where his father was trying to set up business. The free, easy-going social environment of Italy presented a remarkably pleasant contrast to life in Germany. But studies could not be abandoned. He appeared at the entrance test of the famous Polytechnic of Zurich, Switzerland, but failed. So, he entered the high school in the nearby town of Aarau. This was very different from German schools. In particular, it had laboratories, a big garden for botanical work, etc., and students could work there on their own. Albert happily spent one year there, passed the exams, and entered the Polytechnic in 1896.

There were many learned professors at the Polytechnic. However, Albert was often absent from physics and maths classes. The reason : whatever the teachers taught in maths had already been studied by him. In physics he had a strong interest, but the professors' lectures could not fulfil it. He would delve directly and deeply into works of such masters as Maxwell, Kirchhoff, Boltzmann and Hertz. In the laboratory, too, instead of following the prescribed procedures, he would do his own experiments. As for preparing for the exams, a way was found. He had a friend who agreed to lend him his class-notes. Albert poured over these during the last two months, and managed to pass with good marks.

Albert was, by nature, a lonely boy. Since the time he was twelve, he had been thinking about the most profound problems of the world. At school, where boys were occupied with ordinary play, fighting, local racial and religious bickerings, etc., Albert had had no friend. But, at the Polytechnic, there were students from many lands, some of whom would discuss science, history, philosophy, the war-like atmosphere and social injustices. So, several of them became Einstein's close and life-long friends. Among them was a Serbian girl, Mileva Maritsch, whom Albert would often tell about his new ideas and his scientific work. During this entire period, he used to live on a small allowance from his maternal uncles.

Einstein graduated in 1900. With some effort he could get Swiss citizenship too. He could avoid being drafted into the army because of his flat foot. This proved to be a great boon since he had a strong aversion for army and war. So far it was good, but he could not find a job, whereas several of his friends had got jobs at the Polytechnic itself. He kept wandering around for almost two years. Nevertheless, he was not disheartened or unhappy, nor did it affect his sense of humour. His cup of inner contentment was so full that he was not discouraged by adversity, nor elated by good fortune. In a letter to a friend, he put it in these words: "When God created the ass, he gave him a thick skin!"

At last, in 1902, he got a job as a junior officer in the Patent Office at Bern, and married his girl friend Mileva. Though his income was not much, he was happy and contented. In particular, he was very happy about this opportunity to work undisturbed on the problems of physics that had engaged his attention for quite some time. One might wonder as to how he could find the time for research work while doing a full-time job. Einstein wrote in a letter that he used to complete the entire day's work at the Patent Office within three or four hours. Coaxing another friend to join the same office, he wrote, "Eight hours at office, and then, for eight hours, you can do research as you please. And you have the entire Sunday too!" He held the opinion that, in order for one to do basic research, it was better to have an ordinary job than an appointment in a research institution, because the spare time would be totally one's own. Ethically, too, he did not consider it good to be paid for research work as it was supposed to be done for one's own satisfaction.

In Bern, Einstein decided to give tuitions in the evenings - at three franks per hour. He got rather few students and little money. (He used to say that he could have earned more playing on the violin in the streets than teaching physics!). But, in the process, he met two great seekers, Maurice Solovine and Conrad Habicht. Soon, the teacher-pupil relationship was replaced by friendship and the tuition by deep discussions. After the day's work and studies, the three pals would eat together, take long walks, and discuss, smoke cigars and enjoy music concerts. Together they studied many great works of philosophy, physics, literature, etc. They named themselves the Olympian Academy. Einstein cherished the memory of the 'cheerful poverty' of those days all his life.

During this time Einstein accomplished original work in several directions. One was explaining the random, drunkard-like 'Brownian motion' of microscopic particles floating in a liquid (or gas) on the basis of the atomic structure of the liquid. Another was a very simple and elegant explanation of photo-electric effect (the outflow of electric charges from a metal upon shining light on it) on the basis that light consists of tiny bunches of energy called 'photons'. The third one was the Special Theory of Relativity concerned with the structure of space and time. He would often discuss his original ideas with his wife and the above-mentioned friends (and with Michele Besso after they left). Besso drew his attention to many new points. All three works were published in 1905. He also designed a few instruments along with another friend.

The time was just ripe for the advent of the Theory of Relativity. The speed of light had been measured to be the same in various directions, even though the Earth, together with the measuring instruments, was moving at a considerable speed. As no satisfactory and consistent explanation of this could be given, there was an uneasiness in the world of science. Einstein's Theory provided such an explanation easily; therefore, scientists of the entire world came to know about it. Several of them could recognize that this theory marked the beginning of a new era in science.

In 1909 a proposal was mooted to appoint Einstein an associate professor in Zurich University. There was another candidate for the position, Friedrich Adler, Einstein's colleague from the Polytechnic. Adler was highly regarded in the academic and political circles of Zurich, and he was selected. But he declined to accept the position so that the University might benefit from the talent and fame of Einstein.

Later, Einstein went to the German University at Prague as a professor. The environment of social stratification, bureaucratic formality and academic vanity at Prague was not to his liking, but he managed to find and befriend a few outstanding personalities even there. The next year, 1912, saw him back at Zurich—this time in the Polytechnic where he had studied. Around that time, the German Emperor Kaiser Wilhelm wanted to set up a new kind of institute devoted to original theoretical research. Einstein's name was suggested for its directorship. Max Planck and Walter Nernst, famous scientists of Berlin, travelled twice to Zurich to persuade Einstein. He accepted the offer and moved to Berlin in 1914.

His wife Mileva and both sons remained in Switzerland. In fact, strains had been developing in the couple's relationship for quite a while, and these culminated in separation and divorce. (Their younger son Edward underwent mental breakdown in the 1930's and blamed his father for it). In 1919 Einstein married Elsa, his childhood friend, who was also a divorcee.

Whether it was Zurich or Prague or Berlin, Einstein was always surrounded with scholars and students. His lectures on physics were always followed by long discussions, sometimes continuing in a restaurant where he went to eat.

Many scholars of science, mathematics, history, literature and social sciences became his life-long friends. Among them were outstanding scientists like Max Planck and Paul Ehrenfest, as also litterateurs like Romain Rolland. He used to correspond with Rabindranath Tagore and Sigmund Freud regarding the human nature.

In spite of all these friends and discussions, Einstein remained a loner basically. He was always meditating on and thinking about the deep harmony underlying nature. While discussing, he would sometimes lapse into silence and become oblivious of everything. The current of his thoughts was so strong that visitors' conversations or general noises in the vicinity could not impede it, just as a rock or two placed in the vast current of the Ganga hardly affect its mighty flow. In Berlin, once he had to go somewhere with a scientist friend, Philipp Frank. They decided to meet on the Postdam Bridge. Frank thought he could possibly be late by upto an hour, and he was feeling hesitant to have Einstein waste his time waiting on the bridge. But Einstein said coolly, "Thinking is all the work I do. That can be done on the bridge just as well as in my room."

Einstein had to pay in later life for the mathematics classes he had missed out. After the publication of the Special Theory of Relativity, he had been working on Gravitation. He wanted to present gravitation as a curvature of space rather than as a force. In this, Einstein had to seek help from a number of friends and scholars, including prominently the one (Marcel Grossman) whose notes he had used to pass his exams. At last, in 1916, the General Theory of Relativity was published, and in 1919 British astronomers verified it by measuring the curvature of space.

This General Theory was such a new thing that nobody had even searched for it until then. And yet, humanity needed it badly. The War years had seen the whole of Europe littered with dead bodies. The war-weary people were looking for reason and spirituality. The General Theory had a wonderful spiritual effect which transported one away from the tragedy of war into a world of abstract concepts as it connected the gravitation of the Sun and stars with the nature of space and light. And yet, it was not mysticism. Indeed, it proclaimed the victory of rationality over 'self-obvious' notions. Einstein became the most famous scientist of the world by the 20's. Biased opposition from some quarters only added to his fame.

Einstein began to receive invitations from the world over. He travelled all over Europe and U.S.A. with his wife Elsa. Then they toured Japan, and also Sri Lanka, Singapore, Shanghai and Palestine on the way. It was in Shanghai that he received the news of his selection for the Nobel Prize of 1921. Wherever he went, he would lecture on the most abstract concepts of physics. The wonder was that people would come forward in large numbers for these deeply intellectual discourses. In Japan, his first lecture lasted four hours, together with translation. Einstein pitied the people and shortened his lecture in the next city to two and a half hours. He was told, however, that people there were

unhappy as their intellectual stamina had been under-estimated.

Why did the idea of relativity occur to Einstein and not to others? Answering this, he told a friend, "Adults never think about space and time because they have already known about these concepts in childhood. But my mental development was retarded; so I started thinking about it only after I had grown up. Naturally, I could analyze the matter more deeply." What modesty! When he published his General Theory of Relativity, he was confident that it would stand the test of experiment. Again, this confidence stemmed not from a notion of his personal infallibility, but from a faith in the knowability and harmony of nature.

While the world adored him as a hero, he remained a picture of simplicity, oblivious of all the fanfare. He would often travel by the third class on a train, his violin stuck under his arm. Delicacies were served to him, but he would often not even notice what he was eating. He never cared much about the kind of clothes he was wearing. His family and close friends often reminded him about this. When he was to lecture in Vienna, the capital of the Austrian Empire, his hostess, Mrs. Ehrenhaft, took care to get one of his two pairs of pants pressed by a tailor. However, on arriving at the lecture-hall, she saw that Einstein had come wearing the unpressed one!

Einstein was far above personal pride or egoism. Indeed, without rising above the 'self', how could he remain engrossed in the world of abstract concepts and tune himself to the harmonious music of nature? However, this did not make him any less self-respecting and independent. When he applied for a visa for his third visit to U.S.A., an officer of the U.S. Embassy began to question him about his political beliefs. Thereupon, he refused to go to America altogether. Later, in the 50's, when some scientists in U.S.A. were being targeted because of political reasons, he advised them not to give explanations to the government, even if they had to go to jail.

Einstein had many friends, but there were some enemies too. Some were opposed to the theory of relativity – not because of scientific reasons but political ones. For, Einstein being a Swiss Jew, how could his theory be correct in Germany? (The funny thing was that 'nationalist' scientists in England and France were criticizing the theory because he was a 'German'. That was why he was not allowed to lecture at the French Academy of Science in Paris). Besides being a Jew, Einstein was a Zionist. Moreover, he was a pacifist, and he had supported the Bolshevik revolution in Russia. Since the theory of relativity was being seen as a triumph of reason over mysticism, some spiritualists were also unhappy with him. The theory was dragged into such loud controversy that the Swedish Academy could never mention 'Relativity' in its statement for the Nobel Prize.

Now Einstein began his search for the 'Unified Field Theory'. He wanted that all interactions of nature (like electro-magnetic and nuclear) be seen as limiting cases of just one field. To him, this was required by the inner harmony of nature, and this would be a consummation of the theory of relativity. So, he

made this his goal for the rest of his life. However, in the twenties was born another revolutionary theory, viz. Quantum Mechanics. According to this theory, even if one knows completely the interaction between two particles, one can never calculate their future positions and velocities precisely; only the probabilities for various possibilities can be found. Quantum Mechanics appeared to contradict the picture of knowability and objective reality of nature that he had enshrined in his heart all his life. Though he was impressed by the practical success of this new theory, and though he himself had, in a way, sown its seed in formulating light particles called 'photons', he could not accept all his life that Quantum Mechanics reflected the true nature of matter. Gradually, he drifted away from the 'mainstream' of physics research.

Meanwhile, the fever of 'nationalism' was rising once again in Germany. Einstein was a confirmed pacifist. After the First World War, it had appeared to him that, with the onset of democracy in Germany, war-mongering would end for ever. But this was not to be. The League of Nations had, in 1922, set up the Commission on Intellectual Co-operation, and Einstein had been made its member. But he found that the League had neither the inclination nor the ability to restrain powerful nations; so, he resigned the membership of the Commission. Slowly, the Nazis became more and more powerful in Germany. There was political stratification even among scientists. In the autumn of 1932, Einstein went to California Institute of Technology (Pasadena, U.S.A.), and never returned to Germany.

Within a few months of this, Hitler seized the power in Germany, and Nazism and unreason were at their highest. Independent-minded scholars in universities too were selectively removed as part of a 'purge'. Einstein had escaped the clutch of the Nazis along with his family, but his house and yacht were confiscated. His scientific works were burnt in public. When he returned to Europe in the spring, he stayed in Belgium whose queen Elisabeth held him in high esteem. The Einsteins were put up at Le Coq, a seaside resort, and armed bodyguards were posted with them round the clock. Their residence was kept such a secret that even their close friends had great difficulty in reaching them. When Einstein's friend and biographer Antonina Valentin arrived there, she found him immersed in his thoughts as ever. At times he came out with his ironic humour and uninhibited laughs. Miss Valentin showed Elsa an album of enemies of the Nazi rule published in Germany. On its first page was a photograph of Einstein together with his 'crimes' beginning with the theory of relativity, and at the bottom was the remark : "Not yet hanged"!

Einstein was convinced that Nazi Germany would not hesitate to unleash war and destruction. So, he urged free countries of Europe to arrange arms and recruit troops for war. This dismayed some of his pacifist friends including Romain Rolland, but Einstein had realized that peace could not be established just by wishing for it. Later (in 1939) he also wrote the famous letter to President Roosevelt of U.S.A. expressing concern about the German A-bomb programme and urging caution and "urgent action if necessary".

Einstein agreed to join the Institute of Advanced Studies at Princeton (U.S.A.) where there would be no duties for him except research of his choice. Here he continued to strive for the unified field theory for over twenty years. An outline of the theory had been published in 1929, but other scientists were not impressed as it was at a very preliminary stage. In 1950 a new version was published; this, too, was criticized as being unreasonable. Though he was completely lonely, he continued with his effort undeterred. He was sustained by a deep faith that nature was knowable, that the Universe was run under definite laws, and these laws were in perfect harmony with each other. He was totally devoted to furthering the tradition of the Greek philosophers, Spinoza, Galileo and Newton. How far his faith was justified, only time will tell.

During the last few years of his life, Einstein made a considerable effort to save the world from the threat of the atom bomb. He also made a plea for a 'world government', but rulers of the world rejected it. It may be mentioned here that his universalistic beliefs did not detract from his love for his race and nation. In 1921, he had accompanied the Zionist leader Chaim Weizmann to U.S.A. to raise funds for the Jewish University in Palestine. His last writing (incomplete) before his death was a message on the occasion of the Independence Day of Israel.

Einstein had rejected organized Biblical religion early in his life. But he did not consider himself an atheist. Spinoza's philosophy, "Nature is God", made a deep impression on his thought and work. As he wrote to Rabindranath Tagore, the truth was not relative to man but absolute. This was his religion, though he could not prove it.

Einstein was never bothered by the thought of his death. In 1916, when he was seriously ill, Max Born's wife went to see him. She found him discussing his own possible death in an intellectual way. He was talking in such a detached manner that Mrs. Born asked, "Are you not afraid of death?" Einstein replied, "I see myself as a part of everything living. Therefore, I am not in the least concerned with the end of any person's existence". This spirit, which appears to be echoing the Bhagavad-gita's, "*Sarva-bhutam atmanam*", remained with him till the very end. In April 1955, he fell ill. The doctors advised surgery on the gall bladder, but he refused and departed from the world in a calm and contented way. In his will, he forbade any official or religious ceremonies at his funeral, and also any fanfare. Einstein is not among us today, but his contribution towards freeing science and truth from 'self-evident' notions will ever remain.

## Viewing Events from Different Situations

*We introduce the concepts of 'absolute' and 'relative'. It is shown how 'absolute' things (like the directions 'up' or 'East') reveal themselves as relative as our horizons expand. In particular, rest and uniform motion are relative to the observer. This everyday experience is further understood with the help of thought-experiments on board a uniformly moving train. Some prominent results of Einstein's Theory of Relativity are outlined.*

### 1. Relative and Absolute

There are some words that mean the same to everybody; other words may have different meanings for different persons. We could start with an interesting example from grammar. No doubt, you are familiar with *noun* and *pronoun*. Now, the funny thing about a pronoun is that it can have entirely different meanings for two persons. Consider, for instance, this playful dialogue between little Sheela and her elder sister

Neela (Fig. 1).

*Sheela:* Didi, I got home first, isn't it?

*Neela:* Oh, yes, *I* got here first.

*Sheela:* No, not you, but *I*; I arrived first.

*Neela:* I am saying the same thing. *I* arrived first.

*Sheela* (thinks hard): OK, *Sheela* arrived here first. *Sheela* is first.

*Neela* (laughs): That is a clever girl.



Fig.1 : "Sheela and Neela"



As you see, the pronoun 'I' here has exactly the opposite meanings for the two sisters. On the other hand, 'Sheela' (a noun) means the same thing to both. This brings out the difference between 'relative' and 'absolute': that word or quantity that is different for different persons is called *relative*, and that which means the same to everybody is called *absolute*. When we use words which are relative to persons (like I, you, etc.), their meanings can be understood only in relation to the person speaking or hearing them, whereas this problem does not arise in case of absolute words (like Neela and Sheela).

It turns out that this difference between relative and absolute is as important in physics as it is in grammar. Suppose I am situated in Ranchi and a reader is in Delhi, and someone asks, "Which is farther: Chandigarh or Calcutta?" Of course, we both will answer differently. Or, if the question is about the distance to Chandigarh, I would say 1539 km, whereas the reader will say 244 km. Who is right? Actually, each is right in his own way. The answers of both are different because the question concerns something (the distance to Chandigarh) which is defined *relative to the observer's position*.

Consider another example. You must have heard or read about the Science City in Calcutta. Maybe, you have even visited it. It is situated by the side of the Eastern By-pass between Salt Lake and Park Circus (Fig. 2). Can you say on which side of the highway the Science City is situated: right or left? Surely, you will say that the answer is relative to the travelers on the highway. For Suresh coming from the Salt Lake side, the answer is right; for Mahesh approaching from the Park Circus side, it is left. If you want to say it in an absolute way, you may say that the Science City is on the *West* side of the highway.

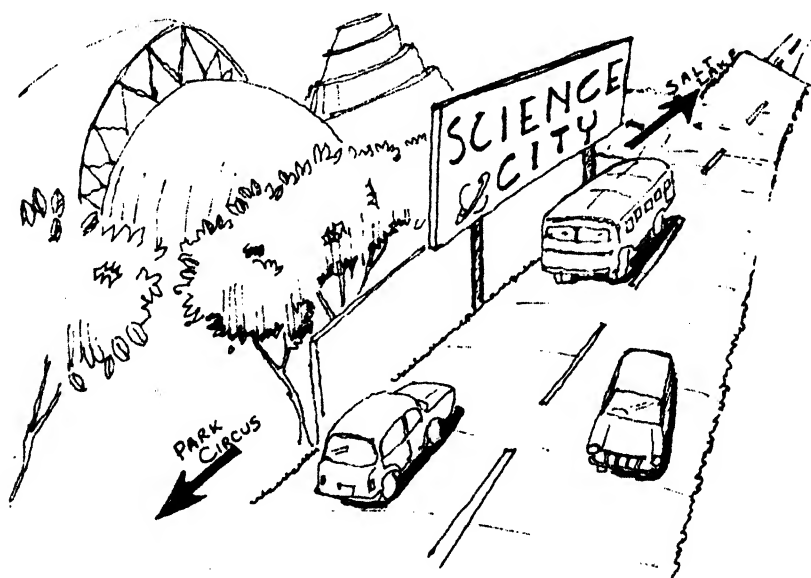
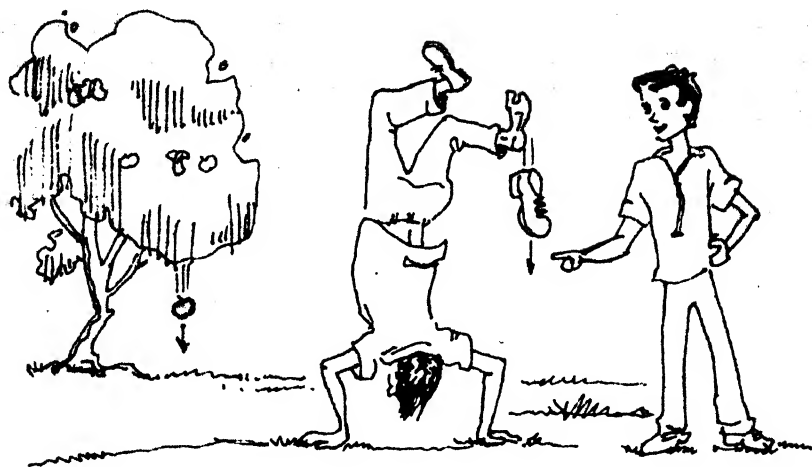


Fig.2 Right or left?



(a)

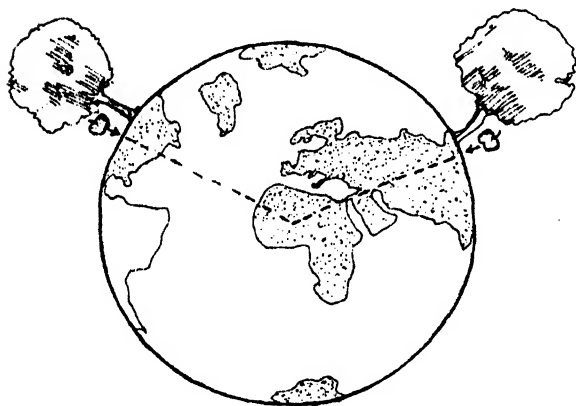


Fig. 3: The definition of 'down':

(a) absolute; (b) relative

(b)

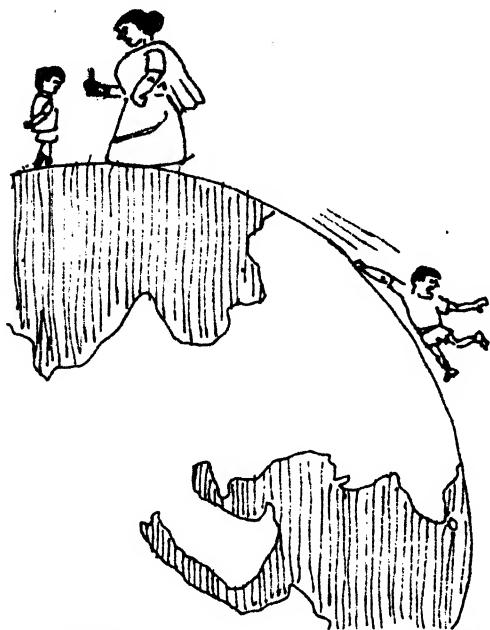
## 2. Absolute Can Become Relative

Now the meaning of absolute and relative is clear, isn't it? You know that a pronoun may be relative, whereas a noun has an absolute meaning. You also know that 'left' and 'right' are relative, whereas 'East' and 'West' are absolute. That is fine, but what can be said about 'up' and 'down'? Are these directions relative to the observer or absolute? If Suresh is standing erect, while Mahesh is standing on his head (as in *Sheershasana*) (Fig. 3a), will it be all right for Mahesh to consider Suresh's 'down' as up?

"No, no", you will immediately say, 'down' is down for everybody". If you think hard, you will perhaps be able to find some good argument in support of this assertion. You might say, for example, "Look here, if an apple from a nearby tree drops, as it reportedly happened in Newton's orchard, it will drop downward. That is, *in whichever direction it drops, we consider that as 'down'*". Mahesh too will have to agree to Suresh's definition. In fact, if Mahesh's own shoe were to slip off from his foot, it too would fall in the same direction.

So, it is seen that 'up' and 'down' are not relative directions but absolute. So far, so good. But, now, what happens if Mahesh and Suresh are not both in the same small orchard, but are separated from each other by thousands of kilometres? Suppose Mahesh Bhai goes away to Mexico, whereas Suresh Bhai

is left behind in his dear motherland India. Will 'downward' direction still be the same for each of them (Fig. 3b)?



**Fig.4:** "Don't go too far from home, or else you might slip and ...."

In olden times, people did not know that the Earth is round like a ball. Far from knowing, they could not even imagine such a thing. The Earth was considered flat – that is, except for hills, ditches, etc. That being so, it was quite natural for them to think that 'down' would be down, as shown in Fig. 3a, whether the observers were near each other or far apart. However, after Magellan's voyage around the world, it became clear that the Earth is round. Therefore, we now realize that the directions of the fall of apples are different in various countries (towards the

centre of the Earth) (Fig. 3b). There are many people even today who find it hard to imagine a round earth, mainly because the notion, of the direction 'down' being the same everywhere, is so deeply rooted in their minds. Such a person, even if he somehow accepts that the Earth is round, may be afraid to venture out too far from his home, being afraid of slipping 'down' from the curved surface of the Earth into empty space (Fig. 4).

Thus, 'down' means towards the centre of the Earth; this direction is not the same in India as in Mexico. Therefore, the directions 'up and 'down', which were considered absolute in olden times, are in fact relative. Moreover, even 'East' and 'West' are not without paradoxes and confusions. For example, for us, Arabia is towards the West. The Arabs say Europe is to their West. For the Europeans, U.S.A. is towards the West, and Japan is even further westwards from U.S.A. Now, if you consider that West of West is West, as common sense suggests, you will conclude that Japan is to the West of India. But actually Japan is towards our *East*!

The above discussion reveals to us that directions like 'up' and 'down', 'East' and 'West', are not really absolute but relative to the position of the observer on the Earth. Note that in day-do-day life, that is, as long as you are

not considering very long distances, the Earth may well be regarded as flat. In that limit, the directions may also be considered absolute. It is a *good approximation*. But things become different when two observers are separated by a large distance. (How large? Comparable to the radius of the Earth.) Then we will have to take into account the roundness of the Earth, and the above-mentioned directions will lose their absolute nature and will become relative. In that case, it will be possible for one single direction to be 'up' for one observer, 'down' for another observer and something else for another one. West of West can become East. If we take into consideration the Earth's rotation about its axis too, we will be forced to conclude that even at the same place on the Earth (say, Delhi), different directions are called 'up' in the morning, evening and night.

Thus, we see that some matters and quantities are absolute and some are relative to the observer. Moreover, things, which ordinarily appear absolute, may turn out to be relative when the horizons of our knowledge expand. Similarly, the Theory of Relativity says: when we consider very fast-moving objects and observers, many of our 'absolutes' become relative. To understand this, let us first consider daily-life experiences of observing things from a moving train.

### 3. Travelling on a Fast Train

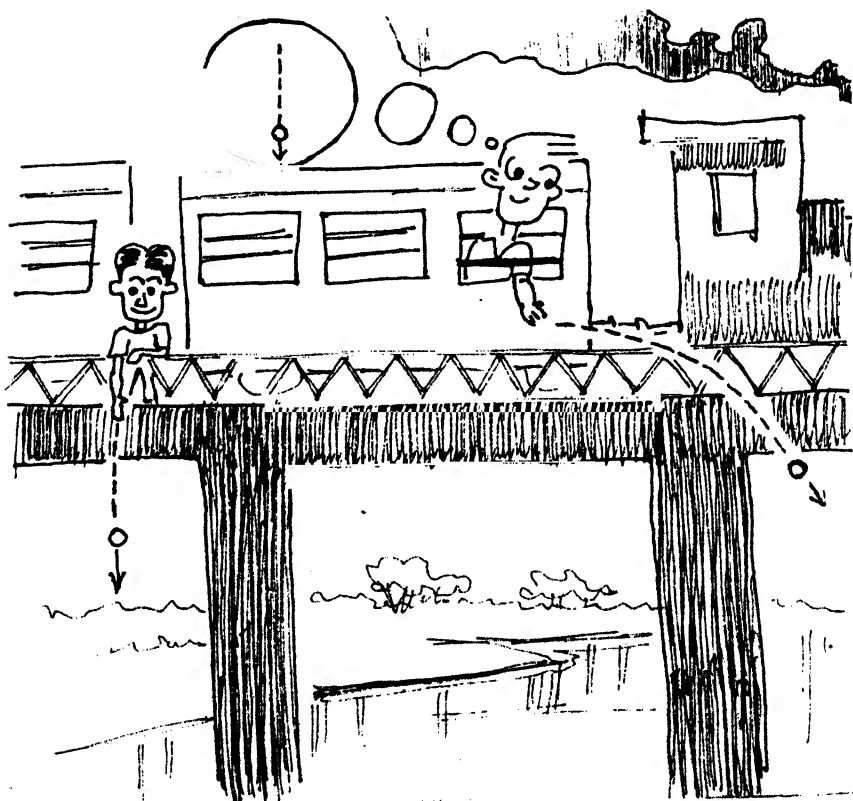
Suppose Mahesh is riding a fast train running southwards, while Suresh is standing on the ground. Mahesh looks out of the window and sees Suresh. He finds Suresh speeding northwards together with trees and pillars and all. Relative to Suresh, on the other hand, Mahesh is moving southwards along with the train. Of Mahesh and Suresh, who should be considered stationary and who should be considered moving? Is there an absolute answer to this question? Perhaps you will say, yes, Suresh is stationary, because he is standing *on the ground*. That only means that Suresh is stationary *relative to the Earth*. But are we sure that the Earth itself is stationary? No. In fact, we know that it rotates on its axis, and revolves around the Sun, and the Sun itself rotates, and .... Then how can we find out who is stationary in a true and absolute sense?

You must have traveled on a train yourself; maybe even on an aeroplane. If you shut your eyes while on a train/plane moving uniformly, how do you feel? Can you feel anything which tells you that you are in motion? Can you feel which way you are moving? Of course not. In fact, it feels the same as if you were stationary. It is only when the train speeds up or slows down, or changes its direction, or the plane is climbing up or down, or undergoes jerks in a storm, that you feel something different. Some people feel sick travelling in a car or ship. This results only from jerks, sharp turns and accelerations, not from uniform motion. Sometimes, it may also happen that your train is standing still on the station. As you look out of the window, another train is beginning to move, but you feel your own train is moving.

Our friend Mahesh too goes through these experiences. We must conclude

that, while there is certainly a *relative* motion between him and Suresh, *it is not possible to say in an absolute way as to who is stationary*. Another thing he can do is to play carrom or a ball game on the moving train and observe carefully how the carrom pieces collide and how the ball bounces and falls. He finds that, as long as the train moves uniformly, the carrom pieces move and collide in exactly the same way as if he were sitting at home and playing. The ball too bounces and moves in its all-too-familiar manner.

Just then another event occurs. As the train crosses the bridge over river Yamuna, a passenger on the train drops a coin from the window with a shout, "*Jamuna maiya ki jai*". Mahesh, looking out of the same window, observes the coin as it falls. He sees that it falls straight downwards (vertically) – just as



**Fig.5: The train passenger sees the coin fall straight down.**

if it had been dropped by someone standing stationary on the bridge (Fig. 5). (Of course, it gets deflected a little by the wind, but there is nothing unusual about it.) So it is not possible to make out the difference between the moving state of the train and a stationary state by such experiments.

If you are still not convinced that there is no difference between uniform motion and rest (being stationary), that is, if you still feel that Suresh, standing

on the ground, is stationary in an absolute sense, then consider this. If Mahesh and Suresh are riding their respective space-ships, each with his own velocity, in outer space, far away from the Earth and even from the Sun, how will each see the motion of the other? Mahesh will say, "I am stationary, but Suresh is moving". Suresh's observation will be exactly the opposite. There is no reason to consider any one of these observations as 'more true' than the other. Both are equally right, just as people in Delhi and those in Mexico City are both equally right in calling different directions as 'down' (Fig. 3b).

#### 4. Einstein's Theory of Relativity

Einstein focussed his attention on observations made by two observers in motion with respect to each other. It had been known for centuries that 'rest'

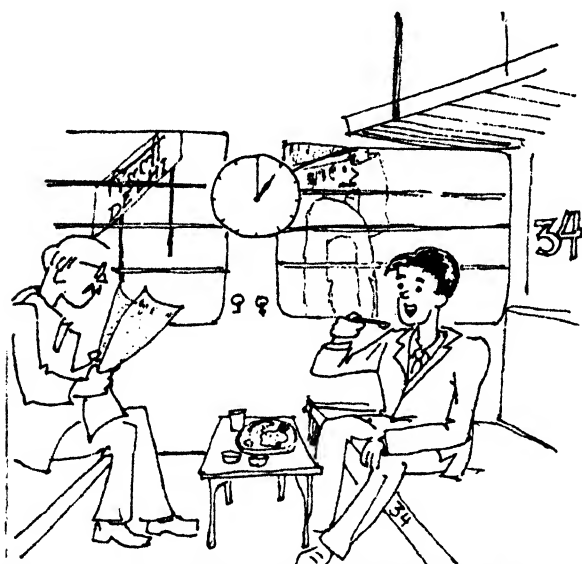


Fig.6: (a) Did the two events (starting lunch and tea) occur at the same place? Yes.

and 'uniform motion' could not be distinguished by any mechanical experiments of the kind described above. Einstein had the insight and courage to proclaim that *no experiment of any kind* (for example, one involving electric or magnetic fields) could establish this distinction. Therefore, he said, a state of 'rest' (no motion) was purely relative. He recognized this as an important and ba-

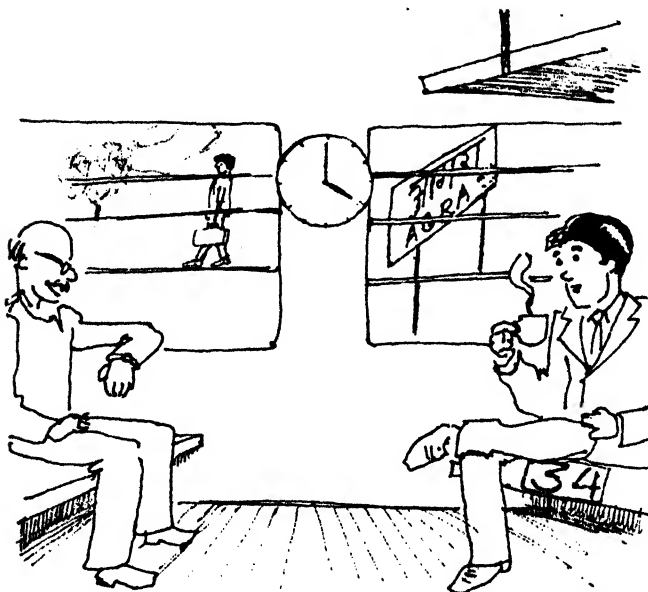
sic law of nature, and showed how it would shake our notions of distances and time-intervals and other physical quantities.

It is very easy to see how space (position of an event) can be relative. Let us once again consider our friend Mahesh riding the south-bound super-fast train. At one o'clock in the afternoon, he is served lunch (Fig. 6a). At four, he takes tea, sitting at the same place, berth no. 34 (Fig. 6b). Of course, *according to all those aboard the train*, he has taken lunch and tea at the same place. However, according to porters standing on the ground and looking through the window, the Saheb started eating lunch at Delhi and took tea at Agra; that is, these two events took place at different points in space. We must conclude, then, that *the notion of two events occurring 'at the same place' is not absolute but relative to observers when these occur at different times.*

Everybody understands this, but *is the reverse also true?* Is the notion of two events occurring 'simultaneously' (at the same time) also relative? Suppose a meteor falls on the planet Venus. Back on Earth, Indian scientists launch a new satellite. Suresh, standing on the Earth, calculates that both events occurred at the same time. But will Mahesh, travelling at a great speed in his space-ship, also agree that the two events occurred simultaneously? Also, if Suresh sees

the two events occurring at different times, is it possible that Mahesh sees the 'earlier' event as later and the 'later' event as earlier, from his moving space-ship?

Einstein's theory of relativity deals with such questions. It is divided into two parts. The Special Theory of Relativity deals with observations made from those reference-frames (or by those



**Fig. 6(b)** Did the two events (starting lunch and tea) occur at the same place? No.

observers) whose own motion is not accelerated. It shows that if two observers (say, Mahesh and Suresh) are moving at speeds approaching 3 lakh km per second relative to each other, then their time-scales are different, i.e. time becomes relative. Not only that, space and time get inter-mingled. For example, the time-interval between two events depends not only on the velocity of the station from which the observation is made, but also on the distance separating the two events. Another startling conclusion of this Theory is that no object or message or signal can ever travel faster than 3 lakh kilometres per second. It also leads to the famous equivalence of mass and energy.

That is not all. According to the General Theory of Relativity, which deals with accelerated reference-frames and gravity, the space around massive stars and galaxies becomes 'curved'. Most of these conclusions are very strange, even hard to imagine. However, as one begins to understand these step by step, one cannot fail to get fascinated. We shall discuss various aspects of both parts of this theory in some detail in the following chapters.

## Two

# **Relativity : Newtonian vs. Einsteinian**

*In this chapter, the concept of 'reference-frame' is introduced and 'space' and 'time' co-ordinates explained. It is shown how a translation or rotation of co-ordinate axes does not change the straight distance between two points or the time-interval between two events. Questions about the nature of space and time are raised, and their answers in the Newtonian scheme are explained. The Newtonian formulation implies certain behaviours of distances, time-intervals and addition of velocities which agree with day-to-day experience. However, the Newtonian velocity-addition, applied to light, contradicts relativity. That brings us to the two postulates of Einstein's Special Theory of Relativity. There is also a narration of how the tremendous velocity of light was measured.*

### **1. Understanding Space and Time**

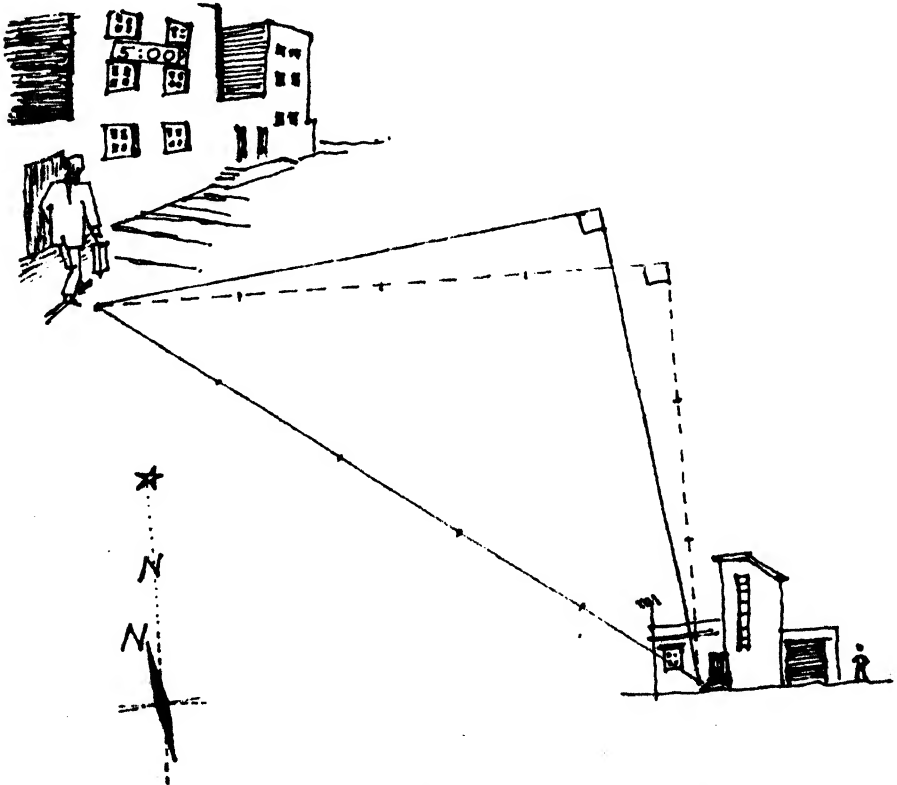
It is not that Einstein was the first one to speak of relativity. Two hundred years before him, Newton gave the laws of motion which imply a certain kind of relativity. The idea of relativity (that all motion is relative) is, in fact, rather ancient. Newton (and Galileo before him) gave a formal shape to it. We have to understand the Newtonian and Einsteinian concepts of relativity and the difference between these. It turns out that these two concepts of relativity are based on different notions of *space* (positions and distances) and *time*. Therefore, we must first consider what space is and what time is.

At this, you might say, "What a question? Even a child knows space and time. Of course, space is ..... well, space is *space*, and time is *time*. *Arre bhai*, it is obvious. It is so simple that it can hardly be explained. All objects are located in space and occupy space, don't they? And all processes or changes proceed in time." But we need to look in more detail at *how* space and time *behave*. In particular, we need to know whether positions of objects and distances between them are the same for all observers (absolute) or different (relative). Similarly, are the times corresponding to the occurrence of various events, and the intervals between events, absolute or relative?



First let us spend a moment on how we make measurements of positions and times. Clearly, our space is three-dimensional. Thus, there are three principal pairs of directions in it: East-West, North-South and Up-Down. Of course, if we confine our attention to just a surface (as an open ground), there are just two dimensions in it and two principal pairs of directions: East-West and North-South. The position of any point on such a surface can be specified by two numbers: one giving the distance of this point from some origin towards the East and the other giving the corresponding northward distance. For example, In Fig. 7(a), if your home is considered as the origin, the position of your office is described by two numbers: 3 km to the North and 4 km to the West. Besides, the clock fixed in your home will give the time of any event. For example, the time of your starting for your office is 9.00 a.m.

Likewise, any observer can describe the position (two numbers, or three if the space is three-dimensional) and time (one number) for any event. For reference, he will have to specify an origin, two or three pairs of axes which intersect at the origin, and a clock. All these together may be called the 'reference-frame' of the observer. Note that the position of any event is relative to the observer's reference-frame. If one observer is sitting in Ranchi and



**Fig. 7: Maps made in two different ways : (a) taking the direction of the Pole Star as 'North'; (b) taking the direction of the magnetic North as 'North'**

another one in Delhi, they will give different numbers to describe the position of any place (e.g. Chandigarh). However, the straight distance between any two points will be the same for both observers.

Now let us see what happens if two reference-frames have the same origin, but *the directions of the principal axes are not the same*. For example, take Fig. 7(a), which shows the positions of your home and office. Suppose two cartographers (map-makers) prepare separate maps of this situation. The first one makes the survey by night, taking the direction of the Pole Star as North, and comes up with the map shown in Fig. 7(a). The other one does the survey work during daylight hours. He does not have the benefit of the stars; so, he takes the magnetic 'North' as the reference; his map is shown in Fig. 7(b). Naturally, the two maps are not identical because the *geographical* and *magnetic* north directions make a small angle. So, in the second map, the northward distance of the office from the home is not 3 km but 3.5 km. The westward distance, too, is similarly found to be 3.57 km instead of 4 km (Fig. 7b). But the interesting part is that the *straight distance* between the two points is  $\sqrt{3.5^2 + 3.57^2} = 5$  km, i.e. the same as in Fig. 7(a). This brings out an important property of space: *the straight distance between any two points is not relative to the system of co-ordinate axes but quite absolute*.

And what about time? Of course, there can be different standards of time too. For example, the clocks in different countries of the world show different readings at the same moment. Suppose you leave your home at 9.00 in the morning. In the afternoon, you leave your office at 5.00. Both these times have been specified as per the Indian Standard Time (IST). In another country, the times for these two events will possibly be different. As per Greenwich Mean Time (GMT), the two events will be seen taking place at 0330 and 1130 hours respectively. However, *the interval between the two events will be the same* (eight hours), no matter which clock is used.

So far, there is nothing unusual. But relativity involves comparison of measurements made by observers moving relative to each other. If measurements are made from a fast-moving space-ship, will the straight distance between your home and office still be 5 km or something else? Will the time-interval between the two events come out the same 8 hours or more or less? Yet another question is: are space and time completely separate from each other? As we have seen above, the two dimensions of space are intertwined, e.g., 'westward' and 'northward' distances can get *mixed up* to some extent if the system of co-ordinate axes is rotated. Therefore, the question asked above can be re-stated thus: is there some curious reference-frame which would convert a space-distance into something like a time-interval?

Even more fundamental questions could be raised. For example, if the house and the office were not there at all, would those two points in space exist, and would the 'distance' between them have a meaning? Here, of course, you will readily say, 'yes'. But wait a minute. What if even the Earth were not there? If the Sun and the Milky Way and, indeed, the entire universe were to

cease to exist, and there were no metre rod with which to make measurements, would the two points still exist, and would the distance between them remain the same (5 km)? If all the clocks of the universe were lost, would time still have a meaning? Would a time-interval be the same for all observers? Note that here we are not talking only of man-made clocks, which are marked 1,2, ... 12 hours, but also the Earth, the Sun, the Moon, etc., whose motions make the basis for measures of time like day, month and year, and also all the atoms

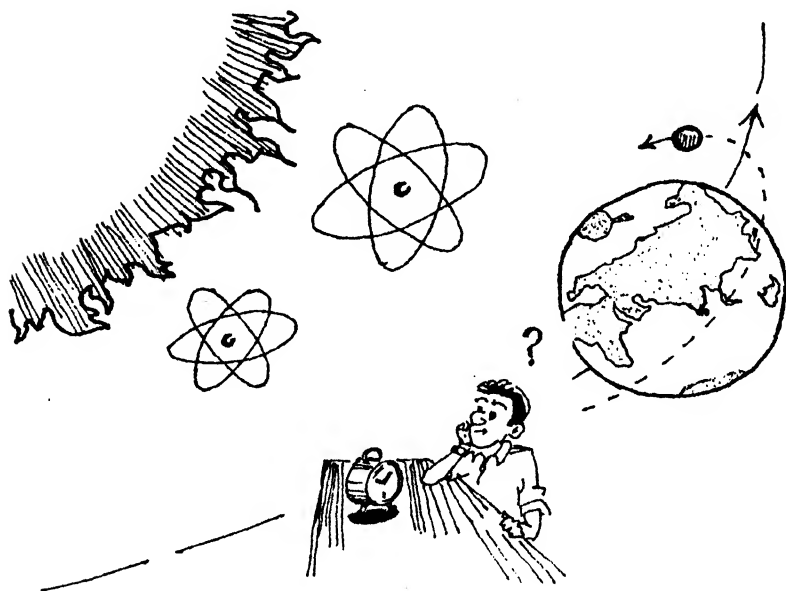


Fig. 8: There are many 'clocks' in nature.

in which electrons go round in definite time-intervals (Fig. 8). If all of these were to disappear, and *if no motion or change of any kind were to take place*, would time have the same meaning and the same rate of flow?

Does the universe have a boundary or a limit? Does space exist and can points be defined outside this boundary? Is the universe limited in time? If so, did time exist before the universe was born, and will it continue to exist after the universe gets dissolved?

## 2. Newtonian Relativity

Perhaps you will find questions of the above kind odd and even meaningless. May be you will say, "Oh, come on, this is not science. Philosophers may spend life-times in arguing such questions, but surely, most of these cannot have any definite answers." Newton, however, realized that laws of motion must be based on well-defined notions of space and time. Therefore, he tried to give clear and definite answers to these questions (without any proof). He

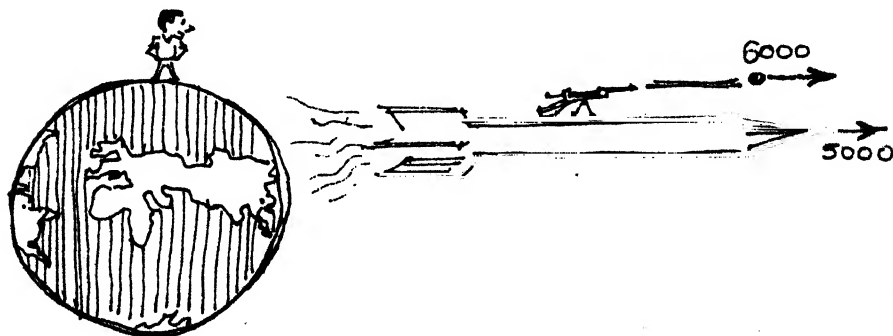
said *space is quite absolute*, irrespective of objects and observers. Space, according to him, is infinite, the same everywhere and fixed. Likewise, *time is perfectly absolute*: it goes on the same way like mathematical counting, whether any events occur or not, whether you and I or any observers are there, whether the Moon and the stars exist or not, regardless of the state of the observer measuring the time. In Newton's scheme, space and time are absolutely different entities, with no scope for one being changed into the other. Nor is there any absolute limit on the speed of objects or signals.

When Newton said these things, he would not have thought that he was presenting a new theory. He was only saying in formal words what everybody 'felt' by common observation. But even in those days, there were some philosophers who could not accept such a nature of space and time as self-evident.

Newton considered the absolute space as fixed, without motion. Therefore, in his scheme, any laboratory or system of co-ordinates is to be considered stationary if it is not moving relative to this space. Any other laboratory, moving at a uniform velocity in this space, is not to be considered stationary. Yet, there is a certain relativity in Newton's laws of motion: these laws are just as much applicable in the uniformly moving laboratory as in the one at rest. We have seen in the last chapter how, in the uniformly moving train or aeroplane, various objects move exactly in the same 'natural' way as on the ground. Thus, we see that Newtonian relativity is consistent with our daily-life experiences.

Now, let us apply these Newtonian concepts to the two events mentioned above: your leaving home in the morning and leaving the office in the afternoon. How would these events be seen from a uniformly moving space-ship? If time flows in an absolute and mathematical way, the time-interval between the two events will remain 8 hours – neither less, nor more. Also, since the space and its distances are absolute, the distance between home and office as measured from the running space-ship will be the same as before, i.e. 5 km.

Let us also investigate how two velocities add. Suppose a spaceman in the flying space-ship shoots a rifle (Fig. 9a). Let us say the bullet is fired from the



**Fig.9(a): Addition of two velocities: parallel**

rifle with a speed of 1000 km per hour (relative to the rifle, of course), and that the space-ship itself is moving at 5000 km per hour relative to the Earth *in the same direction*. Then will the bullet's speed as seen by an observer standing on the Earth also be 1000 km per hour or something else?

You will say, "What a trivial question! Why, since the rifle itself is being carried along with the space-ship at 5000 km per hour, won't this velocity be added to that of the bullet? Thus, the Earth observer will see the bullet flying off at 6000 km per hour."

And if the space-man were to fire the bullet in the direction *opposite to the space-ship's motion* (Fig. 9b)?

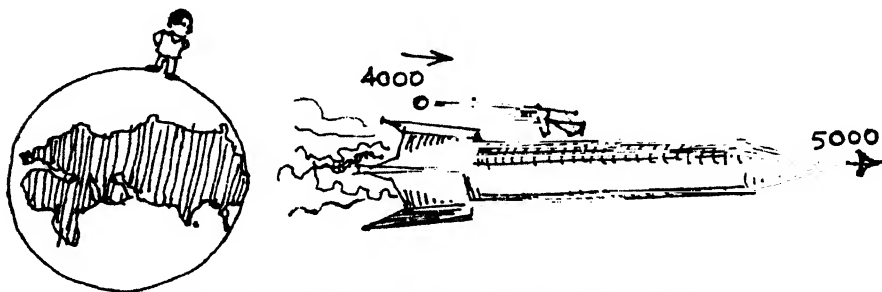


Fig. 9(b) : Addition of two velocities: anti-parallel

"Then the bullet's velocity as measured by the Earth observer will be 4000 km per hour in the same direction as the motion of the space-ship."

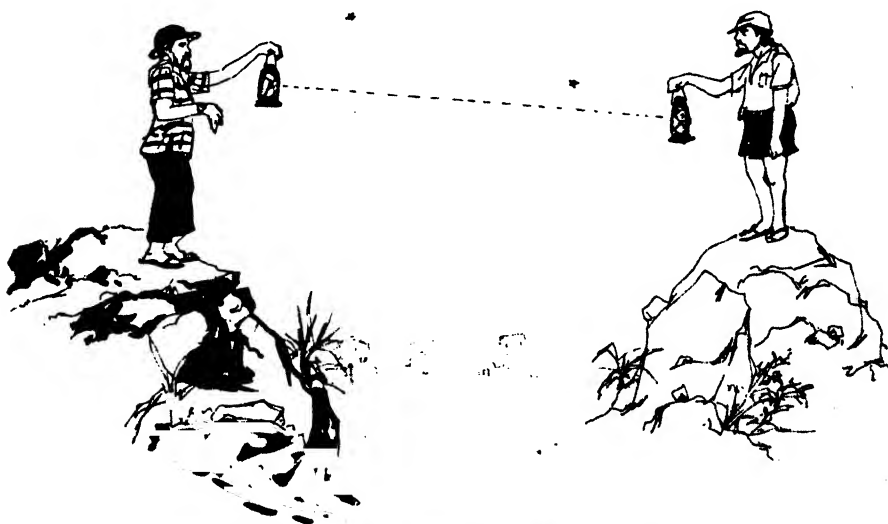
The result of addition of velocities as given by Newtonian relativity is also the same as obtained above from your common sense. That is, the velocity of any object relative to the Earth = velocity of the space-ship itself (relative to the Earth)  $\pm$  velocity of that object relative to the space-ship. Here, the sign  $\pm$  means that the two velocities are to be added together if these are in the same direction and subtracted if the directions are opposite.

May be, by now, you are fed up with all these results of Newtonian relativity. "So, what is new about it?", you might say, "Anybody with a little common sense can see that eight hours are eight hours, that the space-ship's velocity has to be added to that of the bullet, etc.". But it must not be thought that common sense will lead to the right answer *in every case*. Remember, just a few hundred years ago, most people's common sense insisted that 'up' meant the same direction everywhere. However, once sailors circled the entire Earth, and it became possible to compare the directions called 'up' in countries far from each other, common sense was baffled. In a similar way, Einstein brought out another limitation of our common sense. He pointed out that our common sense was limited to velocities much smaller than that of light. He also showed that if our space-ship moved at a velocity approaching that of light, all the above results given by common sense and Newtonian relativity would be overturned. Let us see how it comes about.

### 3. How Fast Does Light Travel?

First let us get some idea of just how fast light is propagated in empty space. You might perhaps think, if you had not been told otherwise, that light gets from one place to another *instantly*. Of course, day-to-day experience supports this notion. When you switch on a torch in the dark, various objects in front, whether near or far, become visible immediately. In olden days, seeing things like this, people thought that light must be moving infinitely fast. But there were some scientists who thought otherwise: could it not be that light traveled much faster than anything else, too fast to be seen moving, *but still not infinitely fast*?

Galileo of Italy was apparently the first one to make a serious experimental attempt to find out the truth. Yes, it was the same Galileo who worked on relativity and also wrote about the Earth going round the Sun. He and his assistant went out on a dark night to two hills separated by a few miles. Each man held a specially designed lantern. It was enclosed in an opaque casing; there was a window which could be opened or closed with a shutter. Thus, light could be let out in any direction by opening the shutter. After both men had taken their respective positions, Galileo opened the shutter of his lantern and let out light in the direction of his assistant (Fig. 10). The assistant had already been instructed to open the shutter of his own lantern as soon as he saw Galileo's light. Thus, Galileo saw light on the other hill a moment after he opened the shutter. He measured, with a clock, the time-delay between opening his shutter and seeing the other light. This delay was rather small and Galileo rightly suspected that a part or whole of it could be attributed to human reflex



**Fig.10: Galileo tries to measure the velocity of light**

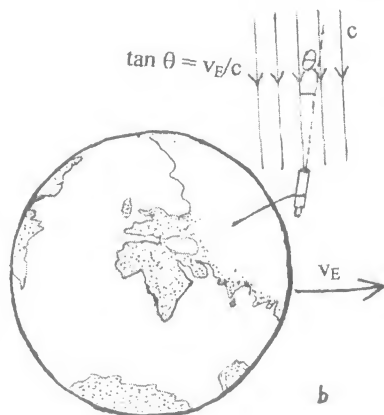
and shutter movement time. So, he repeated the experiment with a larger separation between the observers, and found that the time-delay was the same as before. Galileo concluded, not that light did not take any time to traverse the

extra distance, but that the time taken was too small to be measured in that way.

Later scientists followed Galileo's lead and were more successful. Some of them felt that distances required in the measurement of the huge speed of light could be obtained only in outer space, not on the Earth. Thus, Roemer, a Dutch astronomer, measured times taken by light to travel known astronomical distances. Others measured the velocity of light by relating it to the *aberration* of stars. Aberration means the small angular displacement of the apparent direction of a star from its true direction due to motion of the Earth. It is like a man walking in the rain seeing the rain-drops come at an angle whereas these are actually coming down vertically (Fig. 11a).

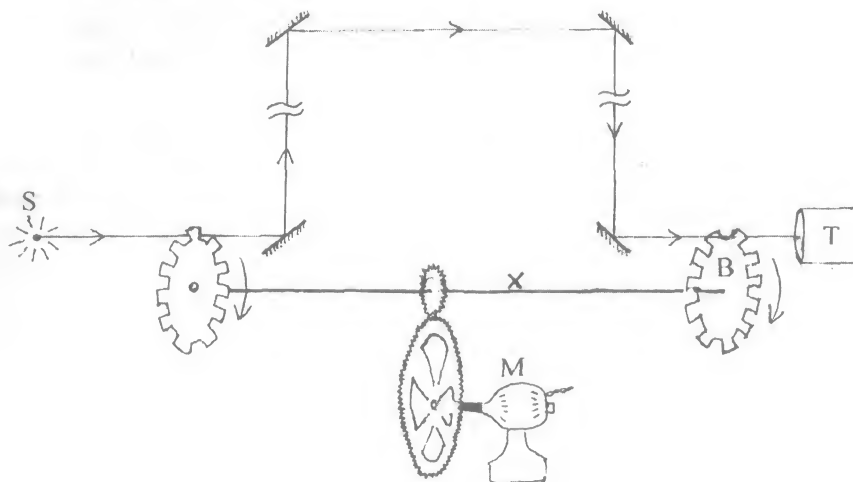
Likewise, if we imagine the Earth running in the space (Fig. 11b), then the light coming from any distant star will be seen coming in a direction slightly different from its true direction. After six months, the Earth will be moving in the opposite direction. Therefore, the apparent displacement of the star will be the other way. Seeing the apparent directions of the star in the two positions, and measuring the angle between them, gives us the aberration angle. The velocity of light can then be calculated from this angle.

Also, many other scientists succeeded in refining Galileo's idea sufficiently to measure the velocity of light by terrestrial experiments. Fig. 12 shows, as an example, the outline of the experimental set-up of the French physicist Fizeau. It consists essentially of two identical wheels, each with many cogs cut into it, both rigidly mounted on the same axle which can be rotated at



**Fig. 11: Explanation of aberration : (a) a man running in the rain; (b) the Earth running in a shower of light**

high speeds. The wheels A and B are mounted in such a way that cogs in the first wheel face openings in the other one. Light from source S is made to pass through an opening in the first wheel, then travel a considerable distance, and finally pass through the rim of the other wheel. If the wheels are stationary,



*Fig. 12: Fizeau's measurement of the velocity of light*

certainly the light will not pass through, as light passing through an opening in the first wheel will run into a cog in the other wheel. Now, the axle carrying both wheels is rotated slowly. Still, light cannot pass. When the speed of rotation is increased continuously, a situation comes when light passes through both wheels and is seen in the telescope T fixed on the other side. How can this happen? Well, the place of the cog of the wheel is taken by the next opening by the time light travels from one wheel to the other; therefore, light passing through an opening in the first wheel encounters an opening in the second wheel too. Knowing the rotational speed, the number of cogs in each wheel and the distance traveled by light, the velocity of light can be calculated.

The best measurements of the velocity of light these days are made electronically or with lasers. In 1964, Alvager carried out an experiment in Geneva in which the time taken by gamma rays (high-frequency light) to cover a known small distance was measured with electronic circuits. In a nuclear reaction called 'pion-decay', two gamma photons were emitted at the same time. One was detected in a detector A near the source, and the other in detector B some distance away. Dividing this distance by the time lag between the two detector pulses, the velocity of photons could be found. In all such measurements, the velocity of light in free space has been found to be as large as 3 lakh km per second.

#### 4. Light and Einstein's Relativity

We have seen already that experiments with motions of objects can never help



us find in an absolute way whether a given reference-frame is at rest or moving uniformly. But what if we do experiments with light? Imagine a space-ship flying at a tremendous speed of 2.5 lakh km per second: this speed is close to that of light. The length of this futuristic vehicle is also staggering: 2 lakh km. It passes by the Earth. As the centre of the space-ship passes by point O on the Earth, a flash of light occurs at O for just a moment (Fig. 13). Naturally, at that moment, light will start spreading out from that point in all directions. Our question is: in which direction will light travel the fastest?

Consider two points A and B fixed on the Earth, at equal distances from the flash-point O but in opposite directions along the line of flight of the space-ship. The Earth observer finds that the light reaches A and B at the same time (due to symmetry); hence, the velocities of light in the two directions are equal. The observer in the space-ship, too, must find light arriving at A and B at the same time, according to the Newtonian concept of absolute time. However, by that time, the space-ship must have advanced with respect to

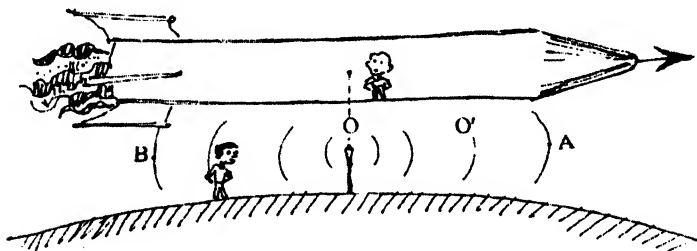


Fig. 13: Is the velocity of light the same in all directions for both observers?

the Earth. Suppose its centre has moved from O to O' during this time. So, our space-man will see that the light has moved a forward distance of only O'A in this time whereas, in the backward direction, a larger distance O'B has been traversed. He, therefore, concludes that light is travelling faster in the backward direction, i.e. in the direction of motion of the Earth.

Thus, the above result enables us to conclude *in an absolute way* that the space-ship is in motion, because we might say, "A lab is moving if the velocity of light measured in it is different in different directions". This is against the basic idea of relativity, according to which all systems, whether stationary or moving with uniform velocities, are equivalent. Then, would you say that *relativity applies only to mechanical events (like motions of balls and bullets), and not to light?*"

Einstein laid down the first postulate that *the principle of relativity is not partially true. It is wholly true.* You may conduct any experiments, whether mechanical, acoustical, electrical or optical, but you can never establish in an absolute way whether your space-ship is stationary or in uniform motion. All that can be said is that it does not have any acceleration; in such a state it is called 'inertial'. Secondly, Einstein's theory says, *in vacuum, the velocity of light in all directions is equal and absolute: 3 lakh km per second.* Therefore,

in the above example, both the Earth-bound observer and the space-ship passenger will find light moving with equal velocities in all directions. It is clear from the above discussion that this will drastically change the Newtonian concepts of time and simultaneity. (The space-man will not see the light reaching A and B simultaneously since  $O'A \neq O'B$ .) Einstein had to choose one of the two: absolute time or relativity. He chose relativity.

Einstein's second postulate, i.e the equality of the velocity of light for all observers, may appear to you as illogical. As we have seen above, the velocity of a bullet depends upon the velocity of the rifle from which it is shot. Similarly, in cricket, a fast bowler runs and thereby increases the speed of the ball thrown by him. Then, would not the motion of the source of light affect the velocity of light?

No, it would not. The reason is that rays of light are not first 'sitting' in the source and then pushed out of it, unlike bullets from a rifle. Light is a wave of electric and magnetic fields. It is a well-established fact that the velocity of a wave depends only on the properties of the medium in which it propagates. For example, sound waves propagate in air at normal pressure and temperature at about 330 metres per second, whatever be their source. Suppose a running car blows a horn momentarily. Two observers are standing one km ahead and one km behind the spot where the horn is blown. Will they hear the sound at the same time (Fig. 14)? Certainly yes, because once the waves are set up in the air, the rises and falls in the density of air will propagate in accordance with the air's own nature, regardless of the horn and the car. In the same way, the velocity of light does not depend on that of the source in any way.

That sounds logical. But then the velocity of light should depend on the velocity of the medium, shouldn't it? For example, in Fig. 14, if a breeze were blowing in the same direction as the car, the breeze would 'carry' the waves with it, and the observer in front would hear the horn before the one who is behind.



Fig.14: Will both the observers hear the horn at the same time?

Indeed, the velocity of light, too, depends on the motion of the medium. This fact has been verified by making measurements on light travelling through water flowing in a pipe. Now, in the situation of Fig. 13, too, if the medium of light waves (whatever it might be) is considered stationary relative to the Earth, the Earth observer will find the velocity of light to be equal in all directions. But, as seen from the space-ship, the 'medium' must be moving at a great

speed. So, how can the velocity of light, as measured from the space-ship, be equal in all directions? And, in any case, what is so special about light that Einstein announced a fundamental law for its speed, and not for the speed of any other waves like sound?

Well, the specialty of light lies in the fact that *it can propagate through empty space*. Sound or other usual waves cannot pass through vacuum. For instance, if a space-man standing on the surface of the Moon shouts to another, or even explodes a bomb, would his companion hear it? Absolutely not, because the space there is empty. Of course, if the explosion were accompanied with a flash of light, the other person would see it, just as we see the Sun and the Moon through empty space. Thus, light propagating in free space, being not associated with any medium, propagates with equal speed in all directions.

Until the last part of nineteenth century, most scientists could not imagine a wave propagating without any medium. Water particles move up and down when waves travel on water surface, air particles' density undergoes rises and falls as sound waves advance through air, but how can a wave move without any medium, without any particles making vibrations? Actually, a light wave is nothing but a pattern of increasing and decreasing electric and magnetic fields. Now, these fields can exist without a medium. If the space between two magnets is evacuated with a pump, for example, don't the magnets still attract or repel each other? Certainly they do. Laws of electromagnetic theory tell us that a changing electric field produces a magnetic field and a changing magnetic field produces an electric field. So, it is no wonder that these can go on propagating, feeding on each other, medium or no medium. That is what makes a light wave.

But the situation was not so clear one century ago. Scientists in general believed in some extra-ordinary medium of light pervading all objects and even 'empty' space. It came to be called 'ether'. It was imagined that the Earth, orbiting the Sun, must be moving through a sea of ether. Efforts were made to measure the velocity of the Earth relative to this ether. The brilliant experiments performed by Michelson and Morley around 1881 showed that this 'motion' did not affect the velocity of light. It was most baffling. Many explanations were given, but each one led to further contradictions. Ultimately, in 1905, came Einstein's Special Theory of Relativity which proclaimed that the velocity of light is equal in all direction in all inertial laboratories\*, whether these are

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\* We are repeatedly making use of words like 'observer' and 'laboratory'. These words are meant only to make the discussions lively. Actually, all we need to specify the position and time of an event is a 'reference-frame', which has been explained at the beginning of this chapter. Readers must not imagine that it is necessary for a reference-frame to have a living, suited-booted observer in it. Also, the use of words like 'laboratory', 'space-ship', etc. does not imply that that reference-frame is limited within some boundaries. Measurements can be made to any distances from a given reference-frame.

stationary or moving relative to 'ether'. After that, there was no need of ether any more. We shall see in the next chapter how this theory of Einstein overturned the old Newtonian notions of space and time.



### ***Einstein the Great Musician***

*Einstein was, of course, very fond of playing on the violin. Besides, he would never refuse to help any charitable cause. Once an organization arranged a music concert to raise funds in a small town of Central Germany. Einstein travelled for many hours to perform there. There was a young reporter from the local newspaper who had come there to cover the event. He asked the lady sitting beside him, "Who is the artist playing on the violin tonight?" The lady exclaimed, "Goodness, don't you know? He is the world-famous Einstein." "Of course", the embarrassed reporter muttered, and busied himself in writing his report.*

*Next morning, the city newspaper carried a big news about the performance of "the great musician Albert Einstein", which sang the praises of his "extraordinary musical talent" and his "unmatched skills on the violin". Einstein and his family laughed their hearts out over it. He would often show the cutting to friends and say, "You say I am a scientist? No, I am a great musician – Einstein the fiddler!" Many years later, when he toured U.S.A., he was presented with a very fine violin made of high-quality wood, among other gifts. Einstein declined the gift of the violin, saying that such a violin should be played only by a great master.*

### *City's 'Gift' to Einstein*

*The Municipality of Berlin decided to present Einstein with a house on his fiftieth birthday. However, the official work was so sloppy that it led to many ludicrous situations. Twice it turned out that the plot that had been selected was outside the jurisdiction of the City. Finally, the Municipality offered to buy any plot selected by Einstein. He and Mrs. Einstein, thereupon, went round the city and finally chose a plot on a little hill outside the city, where nature displayed its beauty rather majestically. The Municipality made an agreement with the owner of the land, and also gave the contract to a company to begin the construction.*

*Those were days when Nazism was on the rise in Germany. When the proposal to buy the land was put before the City Council, some 'nationalist' members opposed it rather vocally. The matter was shelved. Meanwhile, some persons began to portray this whole matter as a scandal, and there was much mud-slinging and counter-mud-slinging. Einstein got fed up. He wrote to the Mayor, thanking him for the offer but adding, "Life is too short for me to adopt myself to your government's slow methods. My birthday is already over, anyway. Therefore, I am afraid I cannot accept the gift of a house from the Municipality."*

*However, construction had already started on that plot. Eventually, Einstein had to pay for both the land and the construction. Still, he and his wife were happy to have a house of their choice. (As luck would have it, they had to abandon the house – nay, Germany itself – within three years.)*

## Three

# Time and Space Play Weird Games

*In this chapter, we begin by explaining the concept of 'simultaneity'. We find by doing a thought-experiment that simultaneity of two events is not absolute if the events are separated in space. In fact, even the time-sequence of two events may not be absolute, though a time-reversal of cause and effect is not allowed. Then we describe a light-operated clock which leads us to relativistic time-dilation (expansion). This, in turn, leads to the phenomenon of contraction of all lengths when observed from a moving system. Finally, it is explained that no object or signal can move faster than light through free space, and that this is consistent with the Einsteinian addition of velocities.*

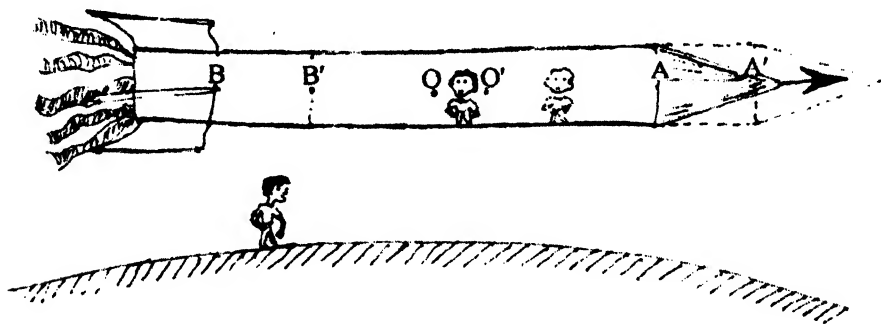
### 1. What Does 'Simultaneous' Mean?

We saw in the last chapter that Newton's relativity says the same things about space and time as we 'feel' by common sense. However, Einstein's theory of relativity assigns a constant and absolute value to the speed of light in free space: 3 lakh km per second. (In brief, we shall call this number as  $c$ .) Whether light is coming from a lamp fixed on the ground or from a spaceship, whether you measure its speed while 'standing still' or running at a very high speed, you will always find it exactly equal to  $c$ . Now, remembering that the speed of light is connected with time and distance, it is natural to expect that fixing this speed will have an effect on the relativity of space and time. Let us see how it happens.

Often we talk about some two events occurring 'at the same time' or 'simultaneously'. For example, suppose an Israeli aeroplane is hijacked and taken to Indonesia. The hijackers demand the release of five Palestinian militants locked up in Israel. Following negotiations, it is agreed that the militants will be released at the Israel-Jordan border and, *at the same time*, the passengers held hostage in the plane will be set free in Jakarta. You will immediately worry that clocks at the two places do not keep the same time. So, let us say the time fixed for both events is 0520 hours GMT (Greenwich

Mean Time, which is a standard for the whole world). All the observers stationary on the Earth will agree on the simultaneity of the two events, no matter what time-zones they are in. But if an observer observes the two events while flying at a very high speed in a space-ship, will he too find the two events occurring simultaneously?

Your common-sense answer will be, "Yes, of course, if the two events are simultaneous, they *have to be* simultaneous for all." However, keeping in mind the contents of the last two chapters, it will not be safe to rely on your common sense completely. The question of simultaneity can be examined more closely in the example taken in the last chapter where a space-ship is flying at a high speed, and a momentary light flash is made at its centre O (Fig. 15). The question is: will the light reach both ends of the space-ship at the same time or at different times? Remember, the space-ship, 2 lakh km in length, is running at a velocity of 2.5 lakh km per second parallel to its length.



**Fig.15: Which end of the space-ship will be hit by light first?**

The astronaut aboard the space-ship will say, "Look here, my vehicle is stationary. (I don't care if your Earth is moving away.) The speed of light in all directions is  $c$  (thanks to Dr. Einstein). The front and back ends are equidistant from the centre. Therefore, the light will reach both ends at the same time."

However, an observer standing on the Earth will say, "How can light reach both ends at the same time? When the light flashes at O, the two ends of the space-ship are at A and B. There is no doubt that light will reach *these two points* simultaneously, its velocity being equal in both directions. But, by the time it reaches these points, the space-ship will have moved to a new position A'B'. Therefore, at that instant, the situation will be that the light has already crossed the rear end, but is yet to reach the front end!

So, we face a contradiction. The astronaut is ready to swear that both events (light hitting the two ends) occur exactly simultaneously, while the Earth-bound observer finds the rear end being reached first and the front end later. In fact, the contradiction can get even worse. What if the light

flashes not at O, the mid-point of the space-ship, but at O', which is just a little distance forward from O? You will easily realize that the astronaut will find the light hitting the front end first, whereas the Earth-bound observer will still find the back end being hit first.

And now listen to the judgement delivered by Professor Einstein. "Both observers are equally right", he announces, smoking his cigar. Indeed, the only way to resolve the above dilemma is to concede that the two events occurring simultaneously according to one observer need not be simultaneous for other observers moving with respect to the first one, as long as the two events are separated in space. In other words, simultaneity of two such events is not absolute but relative to the observer. Moreover, for two events separated in space and time, even their sequence (which one occurs first and which one later) need not be the same for all observers, even if your common sense finds it very odd.

"Wait a minute", you might say at this point. "Common sense may be misleading at times, but what is this business about time-sequence of events being relative? If you mean to say that the first event could be seen as last



**Fig. 16: Which event occurs earlier and which later?  
(a) firing of the air-gun; (b) falling of the bird**

and the last one could be first, I cannot believe it. Just suppose a hunter fires his air-gun at a bird (Fig. 16a), and the bird gets hit and falls down (Fig. 16b). Can any of your high-flying observers say that the poor bird fell first and the bullet was fired later?"

Well, Einstein's theory does not ask you to believe in impossible things like the bullet being fired *after* the victim is hit, or a daughter being born *before* her mother. Detailed calculations with the theory of relativity show



that the sequence of two events is not *always* of a relative character. If the time-separation between the two events is enough for light to travel from the venue of one event to the venue of the other, as seen by any one observer, then the time-sequence will be the same for all observers. In particular, for the cause-effect type of events, that is, where one event is a cause and the other is its effect, the sequence of events is absolute: the cause must be seen as first by all observers and the effect as last. For example, if firing of the bullet is the cause, and the bird getting hit is the effect, then there will be at least as much time separating the two events as required by light to travel from the air-gun to the victim. (Remember, no bullet can travel faster than light!) So, according to the above rule, the time-sequence of these two events will be the same for all observers.

So, now, we can answer the question regarding the plane-hijack crisis. According to the observer stationary on the Earth, the two events (release of the prisoners and release of the hostages) are simultaneous, but these may not be simultaneous for a fast-moving observer since these occur at different locations. Of course, if the hijackers insist that the prisoners be released first and talk to them (hijackers) on the telephone and only *after* that the plane hostages will be released, then a cause-effect relationship will exist, and the sequence of the two events must be the same for all observers. In a nutshell, the relativity of two events being simultaneous ( $t_1 = t_2$ ) or otherwise (say,  $t_1 < t_2$ ) is shown in Table I.

**Table I: Relationship between the times of two events  $t_1$  and  $t_2$**

One Observer O	Other Observer O'	
$t_1 = t_2$	$t'_1 = t'_2$ or $t'_1 < t'_2$ or $t'_1 > t'_2$ (depends on velocity of the observer)	
$t_1 < t_2$ ( $x$ = distance between two events)	If $(t_2 - t_1) \geq x/c$ , then $t'_1 < t'_2$ (Cause-effect relation is possible; sequence is absolute.)	If $(t_2 - t_1) < x/c$ , then $t'_1 = t'_2$ or $t'_1 < t'_2$ or $t'_1 > t'_2$ (Cause-effect relation is not possible; change in sequence is possible.)

It is important to realize that these and other strange results of Einstein's theory of relativity are consequences of the fact that the speed of light is not infinite but 'only' 3 lakh km per second. A few centuries ago, people including many scientists believed the speed of light to be infinite. They also believed that there could be no theoretical limit to the speed with which signals could be sent from one point to the other. But Einstein's theory says that *c*, i.e. 3 lakh km per second, is the upper limit of speed for signals of all kinds – all telephone conversations, television signals.... Even the force due to one body on another (like gravitational force) takes time to 'reach' the other body. In the example of Fig. 15, if the speed of light was infinite, then it would reach the two ends of the space-ship at the same time (instantly), whether seen by

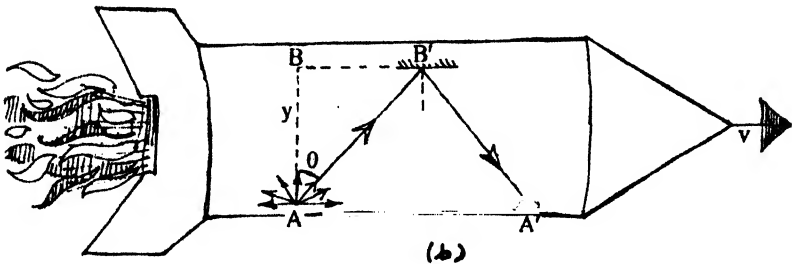
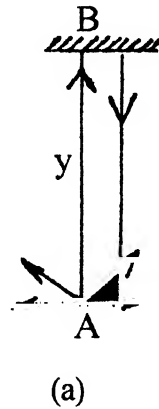
this observer or that. Of course, the value of  $c$  is so large compared to speeds encountered in day-to-day life that it can be considered as practically infinite. Therefore, simultaneity is practically absolute in such situations. The problem arises only when motion of an observer with a speed approaching lakhs of km per second is considered.

## 2. Travel in Space and Stay Young!

It is said that when a friend asked what the theory of relativity was, Einstein replied, "Look here, if you are sitting in a park with your beautiful sweetheart, time passes quickly, but while waiting for her, even a few minutes' time seems like ages."

Perhaps Einstein never actually said that. He did explain, however, that *the rate of ticking of a given clock can be different according to different observers*. If one observer is with the clock, while the other is moving with a large velocity relative to it, the moving observer will find the clock ticking slower. This is not a visual or other illusion. Time itself 'runs' at a slower rate in a system which is moving relative to the observer. How can that be? Let us illustrate it with the following thought-experiment.

Suppose we make a simple clock which works with the help of light (Fig.17a). There is a source of light at A, from where light travels to B, where there is a mirror. Light gets reflected from the mirror and comes back to A. As it reaches A, the clock makes a 'tick'. How long will light take to make this trip? Of course,  $T_0 = 2y/c$ , where  $y$  is the distance between the two points. This time  $T_0$  is the basic unit of time for the clock.



**Fig.17: Construction and working of a 'light clock' :**  
(a) for a stationary observer; (b) for a moving observer

Now, suppose such a clock is lying in a space-ship moving at velocity  $v$  in a direction perpendicular to the distance  $AB$ . The observer aboard the space-ship sees the clock as stationary. So, according to him, light will still take time  $T_0$  in making one round trip. But what will the Earth-bound observer see? He will see (Fig. 17b) that by the time the light reaches up from point  $A$ , the mirror gets shifted from  $B$  to  $B'$ . Therefore, it is light traveling along the direction  $AB'$ , making some angle  $\theta$  with  $AB$ , that strikes the mirror. It gets reflected at the same angle  $\theta$  and reaches the bottom. By that time, the part of the clock, where the source and the detector of light are located, moves from  $A$  to  $A'$ . The speed of light is still  $c$ , but now the distance traveled by light between successive ticks is  $2AB'$ . Therefore, the time unit for the clock is  $T' = 2AB'/c$ , which is greater than  $T_0$ . In other words, the Earth observer finds the clock aboard the space-ship ticking *slower* than the observer does on the space-ship. Just how much slower? It can be calculated very easily. The result is\*:  $T' = T_0 / \sqrt{1-v^2/c^2}$ . For example, if the space-ship's velocity is half the velocity of light, then  $v^2/c^2 = 1/4$ , so that  $T' = T_0 / \sqrt{3/4} = 1.15 T_0$ . This means that, as the space-ship clock strikes 1 sec, 1.15 sec will have passed according to the Earth observer. If the space-ship velocity becomes as much as 90% of the velocity of light,  $T'$  becomes  $T_0 / \sqrt{.19} = 2.3T_0$ , i.e. one year of the space clock is equivalent to 2 years and four months on Earth!

You must not think that this effect is due to the special construction of our clock. According to the theory of relativity, *any* true clock will behave in the same way, whether it is a wall-clock with a large pendulum, the pocket-watch of Mahatma Gandhi or a cheap electronic wrist-watch. By travelling at huge speeds, even atoms will have their electrons slow down in their orbits, your heart-beat will slow down, and so will the graying of your hair. In a nutshell, we may say that it is in the basic nature of time that it slows down in a laboratory moving at a high speed relative to the observer. Thus, we people on the Earth will find that astronauts undertaking long space-flights at very high speeds do not age as quickly as we do. And, believe me, if one could travel as fast as light, time would stop ticking altogether!

Another interesting feature is the *reciprocal nature* of the slowing down of clocks. That is, if the astronaut aboard the space-ship looks at clocks on the Earth, he will say that *these* are all running slow. The reason is that the Earth is also moving relative to the space-ship with an equal and opposite velocity. In Chapter Seven, we shall look at some paradoxes arising out of this fact. For the time being, we shall sum up the result in the following words. If two events occur at the same place in the reference-frame of one observer, then the clock of that reference-frame ('stationary' clock) will show the smallest time-interval ( $T_0$  or 'proper time') between the events. Clocks

\* In Fig. 17b, applying Pythagoras' Theorem to triangle  $ABB'$ , we get

$$AB = \sqrt{(AB')^2 - BB'^2}, \text{ or } cT_0/2 = \sqrt{[(cT'/2)^2 - (vT'/2)^2]}.$$

$$\text{Thus, } T_0 = T' \sqrt{(1-v^2/c^2)}, \text{ or } T' = T_0 / \sqrt{(1-v^2/c^2)}$$

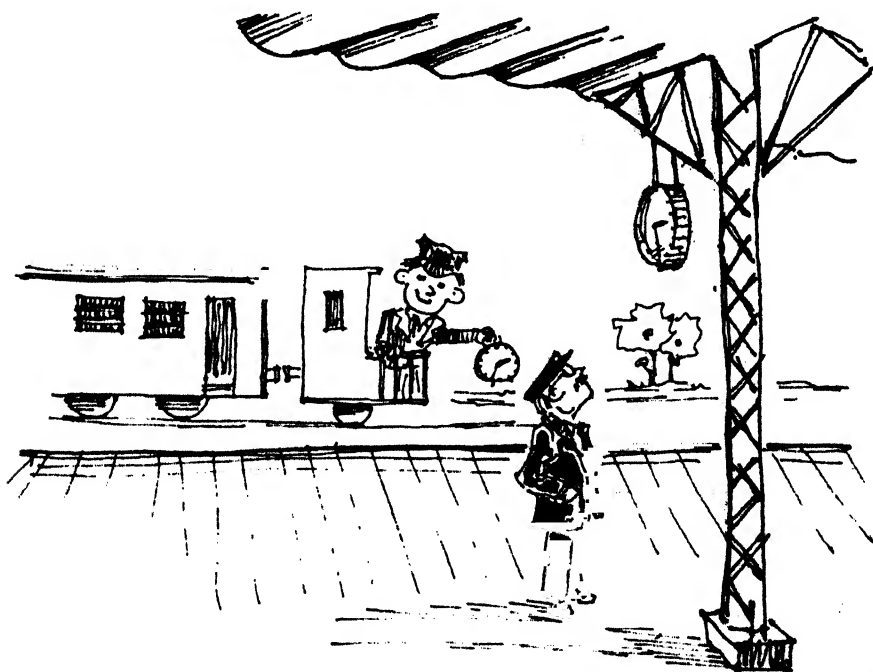
of other observers, in motion relative to this observer, will show a time-interval which is larger by a factor  $1/\sqrt{1-v^2/c^2}$ .

You must have noticed that the behaviour of time, as described above, is against the Newtonian concept of time being absolute and flowing the same way in all situations (Sec. 2.2). A good many years before the theory of relativity was published, all the evidence pointing towards it was available, e.g. the results of the Michelson-Morley experiments. Mathematical scientists were also bothered by the fact that the equations governing electro-magnetic fields (Maxwell's equations) were not consistent with Newtonian relativity. Ad hoc solutions were proposed by many, but none had the courage to abandon the Newtonian notion of absolute time. As we have seen with the help of Fig. 17b, if velocity of light is to be constant, then time must be relative. Finally, Einstein took the plunge, and all pieces of the puzzle fell into a neat pattern.

### 3. How Far Is the Pole Star? Just Ten Km!

If time can dilate (expand), why can't distances change? We shall presently see that any length shrinks when measured from a moving laboratory.

Imagine a space-train running non-stop through a space-station at the speed of 2.5 lakh km per second (Fig. 18). The length of the train is 2 lakh km in the stationary condition, i.e. as measured by passengers aboard the train. Will the Station Master, standing on the platform, also assign the same



**Fig.18: Measurement of the length of a moving train**

length to it or less or more?

Measuring the length of a train moving at such a tremendous speed is quite a problem. But the Station Master has a clever idea. He says, "I know the velocity of the train is 2.5 lakh km per second. All I have to do is to measure the time it takes to cross me and to multiply that time with the velocity."

The Guard on the train also makes a measurement of the same time-interval. Of course, for him, the train is 2 lakh km long and stationary. The Station Master, along with the station and all, is running at a speed of 2.5 lakh km per second. Therefore, according to the Guard, the time taken by the Station Master to cross the train will be  $2.0/2.5 = 0.8$  second. The Guard also notices that the clock fixed on the station wall is running slow because it is moving at such a tremendous speed (as we discussed in the last section). While the Guard's clock measures 0.8 second, the station clock will measure  $0.8 \times \sqrt{1-(2.5/3)^2} = 0.44$  second only. It is clear that the Station Master, going by his own clock, will see the train crossing him in 0.44 second\*. Thus, he will measure the length of the train as  $0.44 \times 2.5 = 1.1$  lakh km!

According to the theory of relativity, the length of *every* object moving at high speed with respect to the observer gets to  $L = L_0 \sqrt{1-v^2/c^2}$  where  $L_0$  is its rest-length, i.e. length measured in a reference-frame in which the object is at rest. Whether the object is made of iron or wood or aluminium does not matter at all. This is different from the contraction of a *material* object moving in a *material* medium due to the *pressure of the medium*. For example, it is found that when a ship moves in water, its length gets contracted slightly due to the water pressing on its front end. That contraction depends on the properties of the material used. A steel boat would shrink less than a wooden one. But, in relativity, the distance between any two points contracts, *irrespective of material*. The factor of contraction is  $\sqrt{1-v^2/c^2}$ , depending solely on the velocity  $v$ . In other words, *space itself contracts*. The various objects in space are like figures drawn on a balloon's surface: as the surface contracts, the figures contract accordingly.

It must be remembered that this contraction in length is also relative and reciprocal. In the above example, the train's length has become less only for the Station Master and others moving relative to it, not for its Guard and passengers. Also, if the train has shrunk for the Station Master, *the Guard sees the entire platform and everything on it contracted*. The relativistic contraction takes place only along the direction of the relative motion, not in other directions. For example, the Guard will see the height of the Station Master unchanged, but his width considerably contracted!

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\* Often a confusion arises as to which clock will run slower: the Station Master's or the Guard's? After all, each of the observers is in motion relative to the other. To get the correct answer, note that the two events (the two ends of the train crossing the S.M.) occur at the same place in the station. Thus, it is the clock fixed in the station that will show the smaller time-interval, according to the rule given above.

Thus, if you wish to travel to the Pole Star (or another star), there is good news for you. The distance to the star may appear enormous as long as you are stationary relative to it. But once you start moving in its direction, the distance contracts. The closer your speed gets to  $c$ , the shorter the distance becomes, in proportion to the factor  $\sqrt{(1-v^2/c^2)}$ . There is no limit to the extent of contraction. If you move sufficiently fast, the distance may even contract to 10 km or less!

The dilation of time and contraction of length described above are not just fantasies of a fertile brain; these have been verified in various ways. A most interesting verification was done with elementary particles called 'muons', which are produced high up in the atmosphere due to showers of 'cosmic rays'. These are unstable particles with a mean life of  $2.2 \mu\text{s}$  (micro-seconds). In an experiment conducted by David Frisch and James Smith, muons having velocity  $0.995c$  were chosen and their intensities were measured (a) at a height of 2000 m, and (b) at sea level. It was found that intensity (b) was 70% of intensity (a). Now, considering that the half-life\* of a muon was  $2.2 \mu\text{s} \times .693 = 1.5 \mu\text{s}$ , and that the time taken to travel 2000 m was  $2000 \text{ m} / .995c = 6.7 \mu\text{s}$ , i.e. more than four half-lives, the intensity of the muons should have been halved more than 4 times. In fact, exact calculation shows that only 5% muons should reach the sea-level. How could 70% of them make it?

The answer lies in time dilation and length contraction. If you view the process from the Earth, the distance is 2000 m but the half-life of the muon is expanded by a factor  $1/\sqrt{(1-v^2/c^2)} = 10$ . Remember, the half-life is  $1.5 \mu\text{s}$  only in the muon's own frame; for us it will be larger. On the other hand, if you view it from the muon's reference-frame, the distance gets contracted by the same factor, i.e. becomes 200 m while the half-life remains the same. Either way, a muon takes only about 0.45 of its half-life to reach the sea-level, which is consistent with the data on intensity reduction.

#### 4. Nothing Can Move Faster than Light!

We have found that a clock kept in a fast-moving vehicle, if seen from the Earth, runs slower than a clock stationary on the Earth. If the Earth-clock shows a time-interval  $T'$ , and the corresponding interval according to the space-clock is  $T_0$ , these are related as  $T' = T_0 / \sqrt{(1-v^2/c^2)}$ . The ratio between the time-intervals recorded by the 'moving' and 'stationary' clocks is  $\sqrt{(1-v^2/c^2)}$ , which is always less than 1. Likewise, the length of any object moving relative to the observer is contracted. If the length of a stationary object is  $L_0$ , then on moving with speed  $v$  the length will contract to:

$$L = L_0 \sqrt{(1-v^2/c^2)}.$$

You must have noticed that, in both cases (time dilation and length

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\* The half-life of a particle is that time in which half of the particles decay. After two half-lives only one-fourth will remain, after three half-lives one-eighth, etc.

contraction), the same multiplier or divider has to be used,  $\sqrt{(1-v^2/c^2)}$ . In the mathematics of relativity, this multiplier appears very frequently. One special property of this number is that if the moving object's speed  $v$  is very small compared to  $c$ , i.e. the ratio  $v/c$  is much smaller than 1, then its square  $v^2/c^2$  will be even smaller, and  $\sqrt{(1-v^2/c^2)}$  will be practically 1. Therefore, at such a small speed, neither time will dilate (expand) nor lengths will contract to any significant extent.

The speeds we encounter in our day-to-day life are extremely small from this point of view. You might think that a supersonic jet plane flies at a great speed. Well, what is its maximum speed? May be 2000 km per hour, i.e. 0.55 km per second. Is this speed very large or very small? In comparison with the speed of light  $c$  (3 lakh km per second), this is practically nothing. You may see by actual calculation that flying at this speed will not result in a reduction of even one millimetre in its length. A clock kept in such a plane will not lose or gain even one second in 24 hours, compared to one kept stationary on the Earth.

Another interesting property is that when the speed  $v$  approaches  $c$ , the value of this multiplier approaches zero. In fact, if the speed of a vehicle



Fig.19: Nature's maximum speed limit

could actually become equal to  $c$ , the clock held stationary in it would come to a complete stop (as seen by us on the Earth); time would become still. And all lengths in it parallel to the direction of motion would vanish. That sounds impossible, doesn't it?

And what if  $v$  were to exceed  $c$ ? Then the value of  $(1-v^2/c^2)$  would become negative, and its square-root would not exist at all. What all these things indicate is that the speed of any object or vehicle or laboratory can never exceed  $c$ , the velocity of light. Rather, it cannot even equal  $c$  (Fig. 19). The fact is that not only an object but even a signal or message of any kind cannot

*travel faster than c.* Even the gravitational or electric or magnetic force of one object cannot 'reach' another object faster than this. For example, if the Sun were suddenly to be torn into pieces by an explosion, we the inhabitants of the Earth would not see or feel it before eight minutes had passed\*. Moreover, the Earth would continue in its orbit around the old position of the Sun for eight minutes.

### 5. Can't Two Speeds Add up to More Than $c$ ?

In the foregoing, we have seen that any object, viewed from any uniformly moving laboratory, cannot move faster than  $c$ , i.e. 3 lakh km per second. That presents us with a fresh problem. Suppose a space-ship moves at a speed of 2 lakh km per second relative to the Earth. If this space-ship has a gun which fires a bullet with a speed of 1.5 lakh km per second in the same direction in which the space-ship is moving, what will be the speed of the bullet *as seen from the Earth*? In Chapter Two, we discussed this problem (Fig. 9) in the context of Newtonian notions of space and time. As per that discussion, the Earth-observer should find the bullet flying off at a speed of 2.0 lakh km per sec *plus* 1.5 lakh km per sec, which comes out greater than  $c$ !

Therefore, it appears that there is an internal contradiction in Einstein's theory of relativity. First it says all inertial laboratories are equivalent, and then it says no object's speed can ever cross the limit of  $c$ . Since the speed of the gun adds to the speed of the bullet, and so on, it should be possible to add up several fractions of  $c$  such that the sum is greater than  $c$ . So, was Einstein wrong, after all?

No, he wasn't. We must remember that at such high speeds, space and time do not have the simple absolute character ascribed to them by Newton. The law of addition of velocities,  $V = v_1 \pm v_2$ , as discussed in Chapter Two, is not valid for such high values of  $v_1$  and  $v_2$ . Simple calculations done according to Einstein's notions of space and time yield a different result:

$$V = (v_1 \pm v_2)/(1 \pm v_1 v_2/c^2),$$

where  $V$  is the 'sum' of two speeds  $v_1$  and  $v_2$ . The  $+$  sign is applicable if  $v_1$  and  $v_2$  are parallel to each other and  $-$  sign if these are oppositely directed. Now we can see that if  $v_1 = 2.0$  lakh km/sec and  $v_2 = 1.5$  lakh km/sec, and both are parallel, as in the above example, the 'sum' of the two will be not  $2 + 1.5 = 3.5$  lakh km/sec, but

$$V = (2 + 1.5)/(1 + 2 \times 1.5/3^2) = 3.5/1.33 = 2.6 \text{ lakh km/sec,}$$

which is *not larger than c*.

The above formula for the 'addition' of speeds has some interesting properties. Firstly, the 'sum' can never exceed  $c$ , in accordance with the

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\* Since the Sun is 150 million km away from the Earth, light takes 150 million/3 lakh = 500 seconds, or nearly eight minutes, to reach the Earth.



theory of relativity. The maximum value it can reach is obtained when both  $v_1$  and  $v_2$  approach  $c$ , and both are parallel, in which case we have:

$$V = (c + c)/(1 + c \cdot c/c^2) = 2c/2 = c.$$

Secondly, if  $v_1$  and  $v_2$  both are day-to-day kinds of speeds, much smaller than  $c$ , then  $v_1/c$  and  $v_2/c$  are very much smaller than 1, and their product  $v_1 v_2/c^2$  is even smaller. So, we can neglect  $v_1 v_2/c^2$  in comparison with 1 in the denominator of the formula, and get  $V = v_1 \pm v_2$ , the familiar Newtonian result.

To conclude, we realize (with a sigh of relief!) that, in all cases where speeds involved are very small compared to the speed of light, Einstein's theory of relativity gives the same results which are obtained from Newton's theory and which appear 'natural' to us bound by our common sense. After all, our common sense is made up of a large number of observations of various events. Now, the events of day-to-day life of most of us do not involve any motions of objects with speeds comparable to  $c$ . Therefore, it is no wonder that our common sense, based on these limited experiences, is bewildered at the 'strange' results of Einstein's theory. Results like simultaneity not being absolute but relative, time intervals and lengths being relative, and speeds not adding up 'as they should' baffle the layman.

This is not unlike the common sense of people in olden times being outraged at the suggestion of a round Earth. After all, for people whose common sense is limited to experiences of places within a 100 km radius of their homes, a round Earth cannot make sense. As one travels large distances, or even hears or reads about far away places, one's common sense expands and gradually becomes comfortable with the notion of a round Earth. Similarly, to a physicist who often deals with large speeds comparable to  $c$ , or even to a reader who reads about such events or makes calculations about these, the above-mentioned 'strange' results gradually become a part of common sense.



### What a Perseverance!

One day Einstein was going to give a bunch of papers to a colleague at his residence. He needed a paper-clip to put these papers together. He searched all over the place but could not find a clip. Then he took a long pin and decided to bend it into a clip. However, he needed a tool to bend the pin properly. So, a search for the tool was launched. He could not find the tool, but managed to lay his hands on a whole box of clips in the process. "Oh yes, this will do", he said. Now, he took a clip, made it into a tool of the required kind, and then proceeded to bend the pin into a clip with its help!

## Four

# Relativity of Mass and Energy

*The relativistic behaviour of space and time has the implication that the mass of a body increases with increase in its velocity. This, in turn, implies that a body can never be accelerated to a velocity  $c$  or more. A simple derivation and the true meaning of the famous equation,  $E = mc^2$ , is given, followed by a fuller explanation to dispel some widely prevalent misconceptions regarding the 'conversion' of mass into energy. Nuclear processes like annihilation, pair-creation, fusion and fission are explained in this light.*

### 1. The Faster, the Heavier!

You must have known doctors advising over-weight patients, "Seth Kaddu Lal ji, or Sethani Bhim Palasi ji, I think you should do some fast walking or jogging to reduce your weight." If the speed of light were not as enormous as it is, e.g. if it were only twenty or thirty km per hour, perhaps doctors would never say such a thing. The reason: one strange consequence of Einstein's theory of relativity is that fast motion of a body actually *increases its weight*! (Of course, 'fast' means having a speed comparable to that of light.)

Let us understand it properly. Suppose you have a '1 kg' weight lying stationary in your laboratory. Naturally, its weight is just one kilogram. However, if you put it in a space-ship and let it fly at a velocity approaching that of light, you, standing on the Earth, may find its weight to be several kg. And why only that piece? All parts of the space-ship and the space-men travelling with it become proportionately heavier. On the other hand, the space-men will find that *your* weight has increased, since, according to them, you are moving at a great speed! It is a different matter that they will find your height or width contracted (Fig. 18), as described in the last chapter.

The multiplier, by which the weight of an object in motion increases, is  $1/\sqrt{(1-v^2/c^2)}$ , i.e. the reciprocal of that by which the length/width decreases. Remember,  $v$  is the speed of the object relative to the observer and  $c$  the speed of light. You can easily calculate that, in order to double your weight

(measured by your doctor), you will have to 'walk' at 2.6 lakh km per second with respect to him, which is 87% of the speed of light. In spite of the greater weight, your width becomes one half of your stationary width, which might please your physician (Fig. 20)!

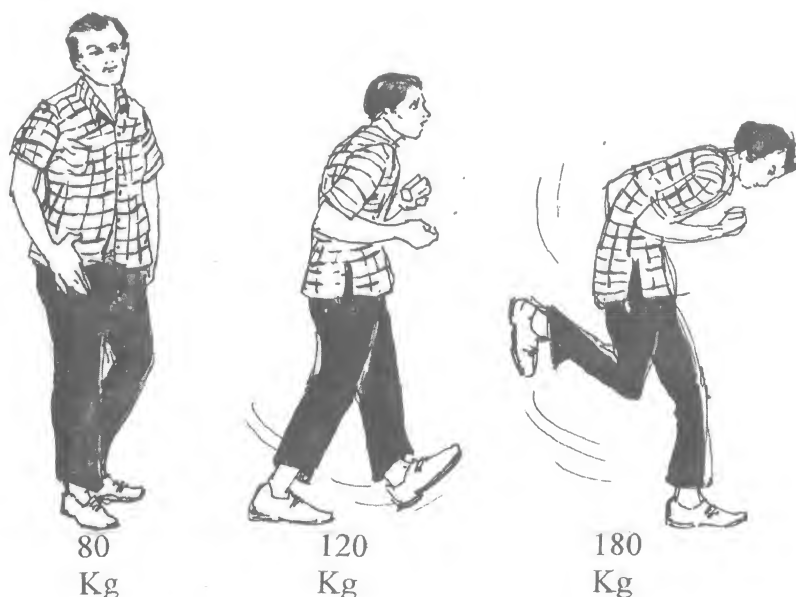


Fig.20: "The mass of an object depends on its velocity". (Einstein)

One clarification may be necessary here. You may have heard that, on the surface of the Moon, the weight of any object becomes only one-sixth of its weight on the Earth. Also, a state of 'weightlessness' exists for any object when it is removed far away from the Earth and other massive bodies. In that situation, there is no such thing as 'falling down'. Such changes in weight, as follow variations in the intensity of gravitation, are different in nature from the relativistic changes described above. When we say that the weight of a body is less on the Moon or in space than on the Earth, we do not mean that the *quantity of matter* in the body becomes any less. There is no change in the properties of the body itself. It is only that the external fields of gravitation are weaker; hence the acceleration of the body in falling under these forces is less. On the other hand, the relativistic increase of weight is a change in an *internal property* of the body called 'mass' in physics, which we have been calling 'weight' for simplicity. When we say that the weight of an object becomes double in motion, what we mean is that it needs to be pushed twice as hard in order to make the same rate of change in its motion. This change in mass is purely relative, as explained above.

One may, of course, ask *why* the mass of a body is more in motion than at rest. Is it a *third* basic element of Einstein's theory, or is it a corollary of the first two? The answer is that it is not an independent postulate, but is

based on the behaviours of space and time already described. Indeed, the strange behaviours of space and time and velocity-addition as described in Chapter Three cannot be true without mass changes. This can be understood with the help of the thought-experiment described below. Only you will have to remember the fundamental law of physics according to which the total momentum (mass  $\times$  velocity) of all bodies taking part in an interaction must remain constant, neither increasing nor decreasing.

Just imagine a bomb exploding into two equal parts by itself (Fig. 21a). If you do not like bombs, you may consider the splitting of a nucleus of mass  $2m$  into two nuclei (fission). The bomb or nucleus is considered stationary before the explosion. According to the law of conservation of momentum stated above, if the masses of the two fragments are equal ( $m$  each), then their velocities must also be equal ( $v$ ) and in opposite directions, so that the total momentum after the explosion is zero as before.

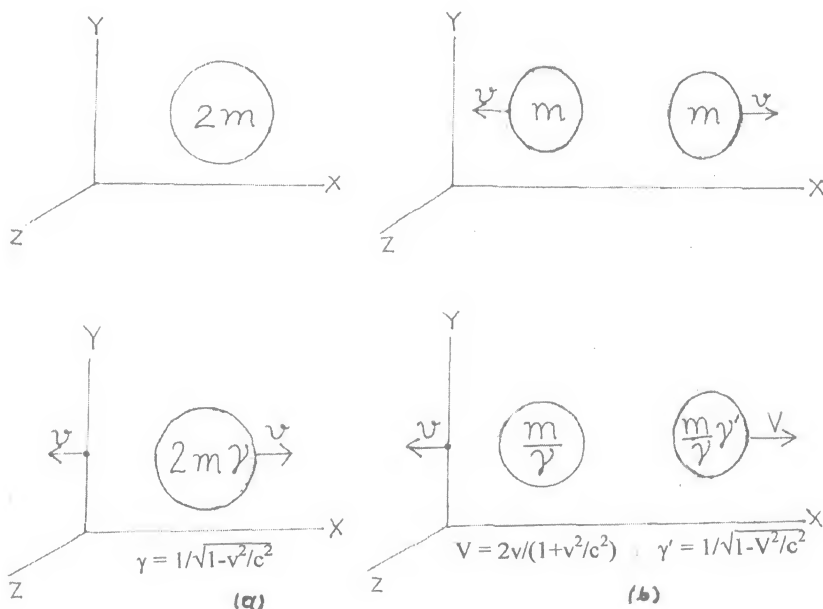


Fig.21: Two views of a bomb explosion : (a) where the bomb is stationary before the explosion; (b) where one of the fragments formed in the

Now, how does the same process appear when viewed from a space-ship flying to the left with velocity  $v$ ? Certainly, the bomb will be found moving rightwards with velocity  $v$  before the explosion, as shown in Fig. 21(b). The left fragment after the explosion will now be found stationary, and the right fragment will move rightwards with some velocity  $V$ . According to the result stated in Section 5 of the last chapter, we obtain

$$V = (v + v)/(1 + v.v/c^2) = 2v/(1 + v^2/c^2) .$$

Now, if all the masses in Fig. 21(b) are taken to be the same as in Fig. 21(a), we shall have to admit that the total momentum before the explosion was  $2mv$ , whereas afterwards it is  $mV = 2mv/(1 + v^2/c^2)$ , i.e. *less than before*. This is a violation of the basic principle of physics stated above. It can be shown easily that this principle will not be violated if we assume that the mass of an object changes with its velocity as stated above and as indicated in Fig. 21(b).\*

## 2. Why Nobody Can Run as Fast as Light

If there is a race between an electron and a photon (a particle of light), which one will win (Fig. 22a)? A photon is always in motion. In empty space, its

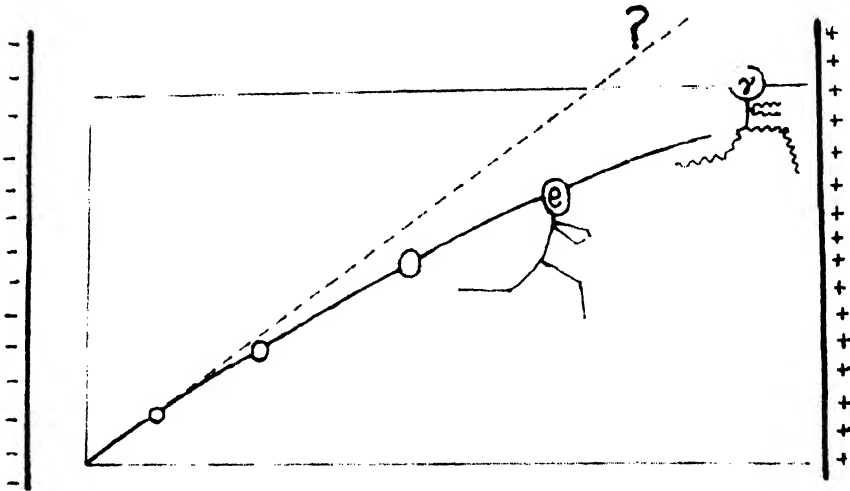


Fig. 22 : Graphs showing speeds of a photon and an electron: (a) photon's velocity; (b) electron's velocity – in the beginning; (c) electron's velocity – full course

\* Fig. 21(a) shows the mass of the initial particle ('rest-mass') as  $2m$  and the masses of fragments as  $m$  each. In Fig. 21 (b), however, the initial nucleus is moving with velocity  $v$ ; so, its mass is not  $2m$  but  $2m\gamma$ , where  $\gamma$  means  $1/\sqrt{1-v^2/c^2}$ . The stationary fragment after the explosion does not weigh  $m$  but only  $m/\gamma$ , because it weighs  $m$  only when moving with velocity  $v$  (Fig. 21a). The other fragment, too, would weigh  $m/\gamma$  when stationary; therefore, at a velocity  $V$ , its mass is  $(m/\gamma)\gamma'$ , where  $\gamma' = 1/\sqrt{1-V^2/c^2}$ . Thus, the net momentum before the event is  $2mv\gamma$ ; afterwards it is:

$$\begin{aligned}
 0 + mV\gamma'\gamma &= m \sqrt{1-v^2/c^2} \cdot \left\{ 2v / (1+v^2/c^2) \cdot \left\{ 1 / \sqrt{1-4v^2/c^2(1+v^2/c^2)^2} \right\} \right\} \\
 &= 2mv \sqrt{1-v^2/c^2} \cdot \left\{ 1 / \sqrt{(1+v^2/c^2)^2 - 4v^2/c^2} \right\} \\
 &= 2mv \sqrt{1-v^2/c^2} / (1-v^2/c^2) \\
 &= 2mv\gamma
 \end{aligned}$$

Therefore, there is no change in the total momentum.

speed is absolutely fixed: 3 lakh km per second. On the other hand, the speed of the electron can be increased with the help of electrical voltages. If we continuously apply a certain fixed electric force on the electron, its speed will go on increasing at a constant rate. So, will its speed not ultimately become larger than the photon's (Fig. 22b)?

Before we can deal with this question, we shall have to answer another one. How do we know that the speed of the electron will go on increasing *at a constant rate* under a constant force? Well, according to Newton's Second Law of Motion, when a constant force is applied on a body, its velocity goes on changing at a constant rate. This rate of change of velocity (acceleration) is obtained by dividing the force by the mass of the body.\*

In the above example, as long as the electron's speed is not very high (i.e. it is much less than  $c$ ), the rate of increase of speed will indeed be constant. However, as its speed approaches a sizeable fraction of 3 lakh km per second, the mass ('weight') of the electron will go on increasing with increasing speed, as per the discussion of the previous section. Therefore, the rate of increase of speed (force divided by mass) will go on *decreasing*, as shown in Fig. 22 (c). The situation is analogous to the following example. If you hit a tennis ball, its speed increases by a considerable amount in a second. But the same force applied on an iron ball, many times heavier than the tennis ball, results in a much smaller rate of increase of speed. For our electron, as its speed gets very close to that of light, the multiplying factor of mass,  $1/\sqrt{1-v^2/c^2}$ , crosses all limits and becomes extremely large. Pushing the 'tiny' electron any further will become harder than pushing the entire Earth. Thus, the speed of the electron (or of any other object) can never equal that of light, not to talk of crossing it.

Actually, the conclusion, that the electron can never overtake the photon, could have been reached even without going into the increase in mass. According to Einstein's theory of relativity, the speed of light in empty space is *always*  $c$ , whether seen from a 'stationary' laboratory or a 'moving' one. Therefore, if the electron has, at some moment, acquired a substantial velocity  $v$  (say,  $0.9c$ ) parallel to the direction of motion of the photon, so what? Mr. Electron himself sees the photon moving ahead at the rate of  $c$ , not just  $(c-v)$ . In other words, in spite of all his great exertions, Mr. Electron, according to his own observation, is losing the same 3 lakh km every second to Mr. Photon. Thus, it is clear that the electron or any other object or signal can never compete with light in terms of speed.

### 3. Mass is Energy (and Vice-Versa)

The mathematical equations of Einstein's theory of relativity are a bit too

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\* The relevant formula here is  $F = ma$ , where  $a$  (acceleration) is the rate of increase of velocity,  $F$  is the force and  $m$  the mass. Actually this formula is not accurate. An accurate calculation will involve some calculus. However, this simple formula is all right for our present purpose.

complicated for the layman. However, there is one equation which almost everyone knows and which will be discussed presently:

$$E = mc^2,$$

where  $E$  is the energy associated with an object of mass  $m$ . Of course,  $c$  is the speed of light in empty space. What it means, in simple words, is that *any object contains an energy* equal to its mass multiplied by the quantity  $c^2$ . Conversely, any energy must be associated with mass (inertia).

This equation is so popular not only because of its mathematical simplicity, but also because it expresses a most profound truth of nature. Indeed, Einstein himself described it as the most important contribution of his Theory. Let us spend some time to understand it clearly.

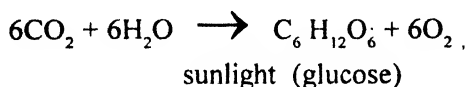
Physics, as you know, is a branch of science dealing with the inter-play of matter and energy. Until about 100 years ago, physicists thought that ‘mass’ and ‘energy’ were two entirely different attributes of objects of nature. Mass and energy were both conserved, but *separately*, as summarized below. Mass could neither be created nor destroyed. Of course, one form of matter could change into another. For example, carbon dioxide and water could change into glucose and oxygen (‘photo-synthesis’) in the presence of sunlight. It came to be known later that it was even possible for one chemical element to change into another, as in radio-activity. However, *the total mass of matter must remain the same* before and after the change. Similarly, energy could never be created nor destroyed, though it could also change from one form into another. In the above example of photo-synthesis, energy of light (radiation) changed into chemical energy of the plant material (glucose). But *the total energy would not change*. In all interactions of various kinds involving matter, radiation, etc., nature was supposed to balance the accounts of mass and energy – strictly and separately.

Einstein’s equation,  $E = mc^2$ , by declaring the equivalence of mass and energy, brought about a merger of both accounts into one. Thus, the separate laws of conservation of mass and energy yielded to the unified law of conservation of ‘mass-energy’. In terms of the fundamentals of physics, this marked a great leap forward. In practice, as we shall see below, it is still useful to keep the two accounts separately for all processes other than nuclear reactions.

As things stand, energies exchanged in various processes in daily life are too tiny to count as equivalent masses. The situation is something like this. If you ask whether money in rupees is equivalent to lands and properties, it is true in an economic sense. However, if a child getting a pocket money of a few one-rupee coins every day asks how much land in the city of Mumbai can be purchased with his savings, the practical answer is ‘none’. In the same way, for ordinary kind of motion, the kinetic energy is not equivalent to any appreciable amount of mass. Even for a jet plane flying at a ‘tremendous’ speed of 1000 km per hour, its kinetic energy is equivalent to

a mass which amounts to less than a ten-billionth of the original mass of the plane. Only with speeds in billions of km per hour can the kinetic energy be equivalent to any appreciable mass.

We may take examples of a few other kinds of energy in daily life and their mass equivalents. Suppose a brick is raised from the ground level to the third floor, i.e. through a height of 10 metres. It gains potential energy as a consequence. The mass equivalent of this energy, however, is smaller than a millionth of a millionth per cent of the original mass of the brick. Again, in the process of photo-synthesis:



the mass equivalent of the energy of sunlight utilized is less than a ten-millionth of the masses of the reactants. The same is true, more or less, of all chemical reactions. In all such processes, therefore, practically no accuracy is lost if we consider the conservation of masses and energies separately. It will be like keeping the account of the child's savings separate from that of the real estate transactions of the family. There is no use of merging these two accounts.

In nuclear reactions, however, the typical energy transactions are so large as to actually count as equivalent masses. For example, if four protons join ('fuse') together to form a nucleus of helium (as happens in the Sun and other stars all the while), the energy released is so large that it is equivalent to 2.8 percent of a proton mass, or 0.7 percent of the combined masses of the reactant protons. In such situations, therefore, old notions of mass and energy have to be

changed. Fig. 23 presents a comparison of typical magnitudes of various kinds of energies (on a logarithmic scale).

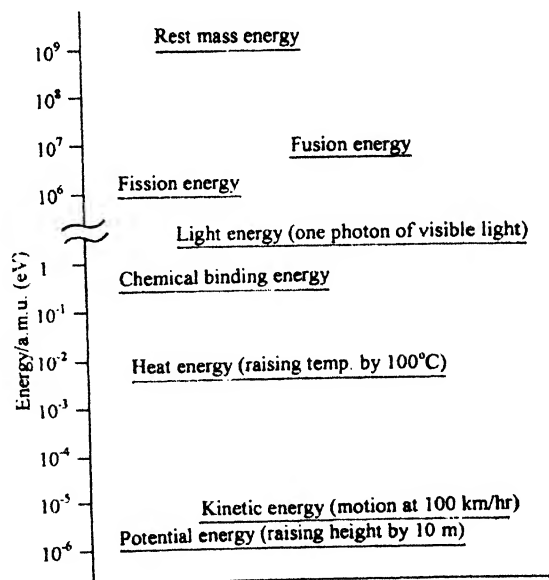


Fig. 23: Comparing typical quantities of various kinds of energy

The relativistic increase in the mass (weight) of a moving object, as described in Sec. 1, is also related to the mass-energy equivalence. Suppose we take a 1-kg object. If it is acted upon by a force for a while so that it



comes to move with a large velocity  $v$ , its mass will also have increased: let us say, it has become 2.5 kg. One may ask where the additional 1.5 kg has come from. The answer is that conservation of mass cannot be realized here without taking into account the kinetic energy. Indeed, the additional 1.5 kg is just the mass equivalent of the kinetic energy (i.e. motion energy) that it has gained. It has arisen from the work done by the force acting on the object.

#### 4. Conversion of Mass into Energy?

There exist several common misconceptions about this 'simple' equation not only among laymen but even among students of physics. For example, many people (and even text-books) say that  $E = mc^2$  relates to a 'conversion' of mass into energy and vice-versa. They say or imply that the separate laws of conservation of mass and energy are not valid because some mass can be changed into energy. So, it is the composite of mass and energy that is conserved. This whole picture is erroneous. Thus, in the above example, where the mass of an object increases from 1 kg to 2.5 kg, is the kinetic energy of the object *converted* into mass? If it were, indeed, then the kinetic energy would have disappeared or decreased. Look, in the conversion of mechanical energy into heat, mechanical energy is *lost*, isn't it? But here, as the mass of the object increases from 1 kg to 2.5 kg, the kinetic energy is not being lost but *gained*. Even in the example of fusion, it is not that mass is being lost. Actually, the four protons form not only a helium nucleus, but also several particles like neutrinos and photons which are moving very fast. They have a significant mass as will be explained later. The kinetic energy of the newly formed helium nucleus too has some mass. When all these masses are taken into account, we see that no mass is lost.

Some persons (and books) also talk of a *conversion of matter into energy* or vice-versa. That too is not correct. Energy is an *attribute* of matter. Of course, it is associated with waves and fields also (e.g. electric field, magnetic field). In any case, an attribute of matter or radiation or field cannot be converted into matter.

The correct position is that both mass and energy are attributes of matter, radiation, etc. Einstein's equation  $E = mc^2$  says that *mass and energy of a system change together, in the same proportion and the same way* (i.e. increasing or decreasing). It would not be a great distortion to say that *mass is energy and energy is mass*. Only the units are different. Wherever mass exists, energy exists. In the above example, as the mass of the object increases by 1.5 kg, its energy (kinetic) also increases. Moreover, when it is stationary and has a mass 1 kg, even then it has an energy proportional to its mass. This is called 'rest-energy'— a kind of concentrated internal energy. Conversely, where energy exists, a proportionate amount of mass exists, as discussed below.

Consider a photon (a speck of light or e.m. radiation). Certainly it has some energy, equal to  $h$  (Planck's constant)  $\times v$  (frequency). Does it, therefore,

have a mass? Traditionally, it was thought that light or radiation did not have mass. But the above equation implies that, yes, *radiation too has mass*. How much? Of course, each photon has mass  $m = E/c^2 = hv/c^2$ . So, it must have inertia of this magnitude. It also has weight in a gravitational field. (According to Einstein's Principle of Equivalence, mass in an inertial sense and mass in a gravitational sense mean the same thing: see Chapter Six.) Thus, the distinction between matter and radiation has been made less sharp. Of course, the distinction has not entirely disappeared. Photons *always* travel with velocity  $c$  in empty space, while matter cannot. What about the rest-mass of a photon? We find that:

$$m_0 = m \sqrt{1-v^2/c^2} = 0 \quad (\text{because } v = c).$$

Thus, *a photon has zero rest-mass* whereas other particles like electrons, protons, etc. have some mass even at rest. (You should not try to visualize a photon at rest because *it cannot be at rest*. "Rest-mass is zero" only means that its energy is entirely due to its motion.)

Thus, mass and energy, two important attributes of matter or radiation, traditionally considered very different, were brought very close – virtually merged – by the equation  $E = mc^2$ . Indeed, it is common practice to specify the rest-masses of elementary particles like electrons in energy units. A stationary electron is said to have a 'mass' of 0.511 MeV and a stationary proton 938 MeV. Actually, these are their respective rest-energies. (One MeV is equal to the kinetic energy that an electron gains or loses as it moves through a potential difference of a million Volts.) In applying the law of conservation of energy, therefore, we shall have to consider rest-energies of matter as well as other forms of energy. Similarly, if we use the law of conservation of mass, then all masses, even those of kinetic energy and radiation, must be included. Both laws are equivalent.

Having debunked several false notions of 'conversion', let us now describe a real conversion in which the equation  $E = mc^2$  is relevant. In 1932, a new fundamental particle was discovered in showers coming from the outer space called *Cosmic Rays*. It had one unit of positive charge and the same rest-mass as the electron. It was called a 'positron'. In fact, this particle was closely related to the electron, as had been predicted already by P.A.M. Dirac. Now, one of the most remarkable properties of the positron is that it can combine with an electron to give – guess what? Nothing at all!

Well, it cannot be entirely nothing. What we mean is that the final product contains no matter, only radiation (photons), as shown in Fig. 24(a). The total energy carried by the photons is equal to the total rest-energy of the two original particles,  $2 m_0 c^2$  (plus their kinetic energy, if any). The same equality can also be stated in terms of mass. Thus, it turns out that *matter can be converted into radiation*. Of course, there are restrictions which govern this conversion called 'annihilation'. Only two particles which are 'opposites' of each other can undergo annihilation. A proton, too, has its opposite (called 'anti-proton'). So, proton and anti-proton may annihilate to produce radiation.

Similar is the case of neutron and 'anti-neutron'.

The reverse process too has been discovered; it is called 'pair-creation'. A high-energy photon can, under the influence of suitable electric fields, convert itself into an electron-positron pair (or any of the other pairs of opposites mentioned above). This is shown in Fig. 24 (b). According to the

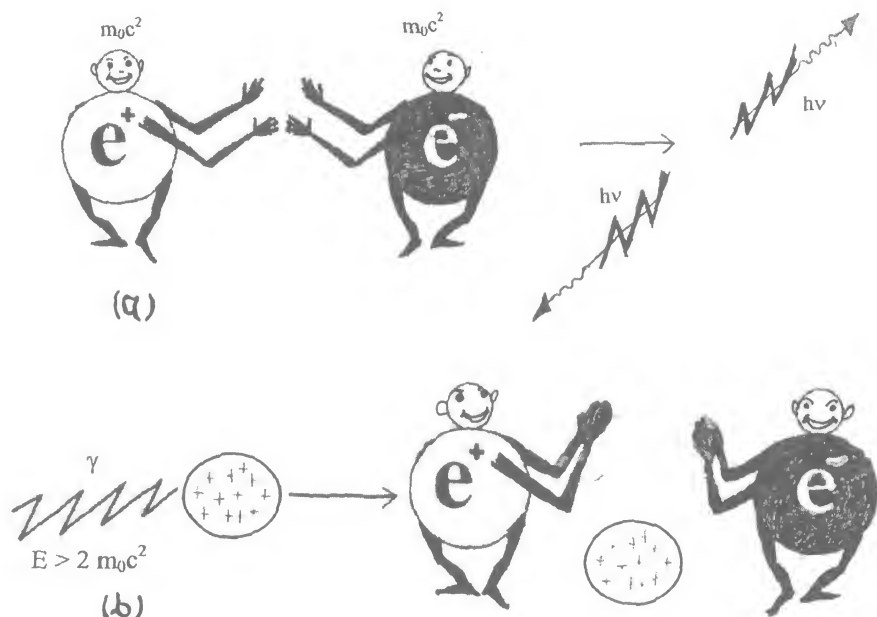


Fig.24: Converting matter and radiation into each other : (a) annihilation;  
(b) pair-creation

principle of conservation of energy, the minimum photon energy needed for the creation of one electron-positron pair is  $2 m_0c^2$ , i.e. 1.022 MeV.

To end this section, let us see just how Einstein arrived at this wonderful equation. Consider an atom at rest (in our laboratory). At a certain moment, it gives out two identical photons (particles of light), in mutually opposite directions (Fig. 25a). According to the photon theory of light (also given by Einstein), if photon has an energy  $\epsilon$ , it has momentum  $\epsilon/c$ , parallel to its direction of motion. Clearly, in Fig. 25(a), the momenta of the two photons cancel each other, and the atom remains at rest after the emission. So, the total momentum is zero before and after the process. The energy of the atom, of course, must decrease by an amount  $2\epsilon$ . By viewing this process from a moving laboratory (say, a space-ship), having a huge velocity  $v$  perpendicular to the directions of the photons in the Earth-based laboratory (to the left), and applying the law of conservation of momentum, we shall be able to see

how the mass of the atom changes.

As seen from the space-ship (Fig. 25b), the atom moves rightwards with a velocity  $v$  both before and after the emission. Since the atom sees both

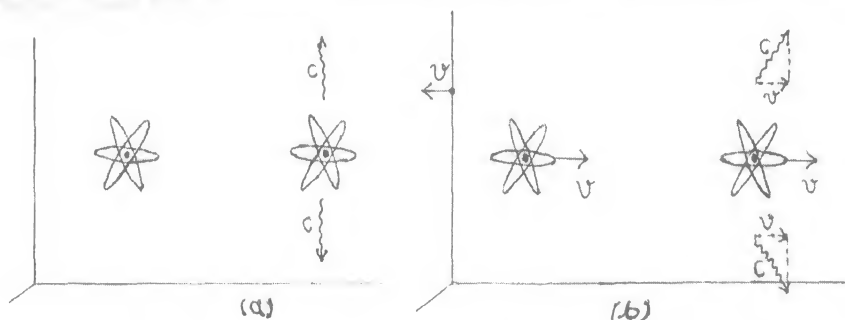


Fig.25: Two views of two-photon emission by an atom : (a) in the reference-frame of the atom itself; (b) in a moving reference-frame

photons travelling in opposite directions, these photons must keep up with it in its rightward motion. In other words, each photon must have a rightward velocity *component* equal to  $v$ ; so the two photons together have a rightward momentum equal to  $(2\varepsilon/c)(v/c)$ . The total rightward momentum must stay as before; therefore, the atom's rightward momentum must *decrease* by this amount. But there is no change in its velocity; hence *its mass must have decreased* by this amount divided by  $v$  (since mass = momentum/velocity), i.e. decrease in mass =  $(2\varepsilon/c)(v/c)(1/v) = 2\varepsilon/c^2$ .

Thus, we conclude that the change in mass of the atom is related to the change in energy by the relation:

$$\Delta m = \Delta E/c^2, \text{ or } \Delta E = \Delta m \cdot c^2.$$

It turns out that this is true for all other kinds of changes in energy, too. For example, if an object is pushed by applying a force, and it gains energy of motion (kinetic energy)  $\Delta E$ , then its mass will also increase by an amount  $\Delta E/c^2$ . Now, if we start with nothing at all (zero mass and zero energy), and gradually build up a particle of mass  $m$  (bringing matter from outside), then a proportionate amount of energy will have come in, whose total amount will be  $mc^2$ . Thus, we have proved that if mass  $m$  is associated with energy  $E$ , then  $E = mc^2$ .

## 5. Nuclear Energy (Atomic Energy)

We have mentioned above that four protons can undergo *fusion* to form a nucleus of He-4 (an isotope of helium), releasing a large amount of energy. The actual sequence of fusion reactions in the sun, shown in Fig. 26(a), reveals that nuclei of carbon and nitrogen are also involved, although there is no net consumption of these. The energy released is over 28 MeV per helium nucleus produced.

It turns out that virtually any light nuclei can release energy by combining (fusing) into somewhat heavier ones. Scientists have tried to make systems for release of fusion energy on the Earth. They have succeeded in making large-scale systems which liberate fusion energy in a violent burst (called

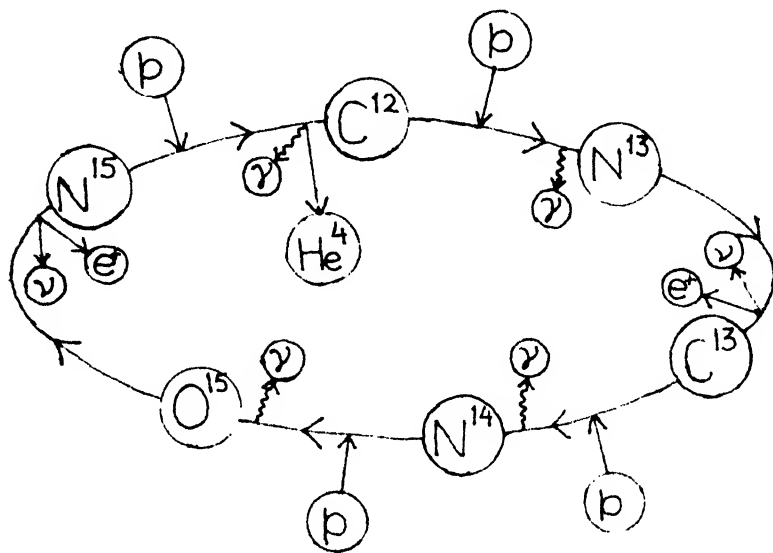
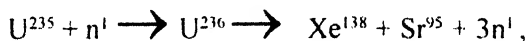


Fig.26(a): A fusion reaction

'hydrogen bombs'), but not systems which can do this in a controlled way (fusion reactors).

Breaking up very large nuclei can also be accompanied with liberation of energy. When a very heavy nucleus like that of Uranium breaks up into two medium-heavy nuclei, for example:



a large amount of energy is released (Fig. 26 b). This energy, expressed in units of mass, amounts to nearly 0.1 percent of the mass of the original nucleus undergoing 'fission' (break-up). Therefore, the combined rest-mass of all the products will be that much less than the rest-mass of the nucleus undergoing fission ( $U^{236}$ ). The so-called atom bomb uses this fission-energy; so do all the 'atomic' reactors producing a significant fraction of electricity in India and several other countries.

In this way, we find that, whereas the mass-energy equivalence is of no importance in ordinary mechanical, chemical and other processes, it manifests itself clearly and noticeably in nuclear processes. As pointed out already, the processes of fission and fusion cannot be said to illustrate a conversion of mass into energy. The 'mass-defect' (difference between the *rest-masses* of products and original nuclei) is made up by the mass equivalent of the energy

released. Of course, even after the fusion/fission, the product nuclei contain a much, much larger energy (rest-energy) stored in them. But that energy is locked up, not available to be released readily and used.

Now, is there any *conversion of matter into radiation* in these processes? The answer is 'no'. The nucleus of any atom consists of smaller particles of two kinds, protons and neutrons. Both are fundamentally similar and are

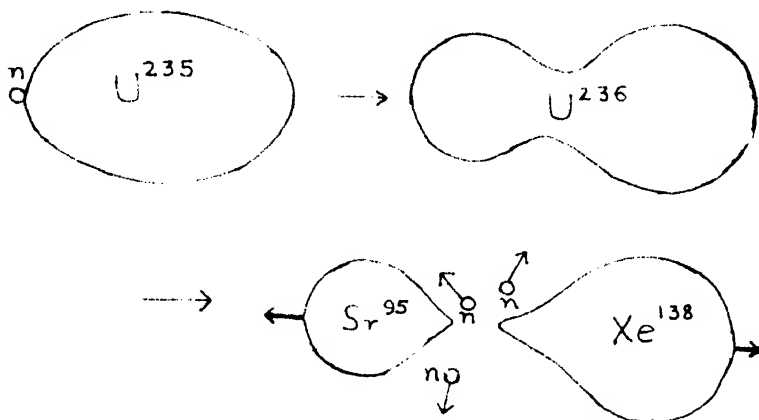


Fig.26(b): A fission reaction

together called 'nucleons'. In nuclear reactions, these nucleons are neither destroyed nor created. Only some protons may change into neutrons or vice-versa, besides re-grouping of nucleons. For example, you may count the total number of nucleons on each side of the fission equation: it comes to 236 on each side. So there is no loss of matter: if 'matter' is taken to mean the building material of nuclei. But, then, how is so much energy released in these processes?

The answer is that there is a tremendous *negative energy of interaction* between various nucleons in the nucleus. (The interaction energy is considered negative because the tighter the nucleons are bound together, the *less* is the total energy of the system. This is the same as saying that *more energy is required* to break up the nucleus into individual nucleons.) Now, a given number of nucleons can be arranged together in many different alternative ways. One arrangement may have smaller interaction energy (greater negative energy) than another kind. Therefore, a change from one arrangement into another may require a huge energy from outside or release energy into the outside world. If we have a large number of nucleons, it turns out that arranging them into medium-mass nuclei (about 60 nucleons each) results in the lowest amount of energy (largest negative interaction energy). Arranging the same total number of nucleons into much smaller or much larger nuclei involves larger energies; thus, these are less stable. Therefore, energy is released by fusion of very small nuclei as well as by fission of very large ones.

This may be explained in a little more detail. The interaction energies stored in nuclei may be divided into many parts. Those parts, which change with changing arrangement, are of two main kinds. There is the positive energy due to the mutual electric repulsion of protons called 'Coulomb energy'. Then there is the positive 'surface energy' associated with the surface area of each nucleus (as with the surface of a liquid drop). Now, in the fission process, as a large nucleus breaks up into two medium-sized ones, the surface energy increases as the total surface area becomes larger. However, the Coulomb energy decreases as the two positively charged product nuclei move away from each other. (see Fig. 26 b). Thus, in fission, the nucleus loses a lot of Coulomb energy. A part of it is utilized to increase the surface energy of the system; the rest is released as kinetic energy and radiation. Similarly, in fusion, the surface energy of the system is greatly decreased. A part of it goes to build up Coulomb energy, while the rest is released.

Thus, it is clear that fission and fusion involve a change of Coulomb/surface energy into kinetic energy and radiation, *without any loss of matter (nucleons)* just as changes in energy levels of atomic electrons result in release of energy (light, etc.) without any loss of electrons.

Sometimes, the credit for the discovery of 'atomic energy' (nuclear energy) and the 'atom bomb' is given to Einstein and his equation  $E = mc^2$ . This should not be taken to mean that fission and fusion would not have been discovered without this equation. However, in a way, giving such a credit is correct. As scientists made accurate measurements of masses of various nuclei, they became aware of small changes in mass that would accompany various nuclear reactions. Einstein's above equation then disclosed the enormous amounts of energy to be released in nuclear reactions like fission and fusion.

Suppose you wish to find the energy released in the neutron-induced fission of the heavy nucleus U-235 into two smaller nuclei and three neutrons. All you have to do is to find the masses of all concerned kinds of nuclei (including the neutron) very accurately using tables given in books. Then the difference between the total mass of the reactants (U-235 nucleus plus a neutron) and the total mass of the products, called *mass-defect*, can be calculated. This, multiplied by  $c^2$ , is the amount of energy to be released in the reaction. Similarly, the energy evolved in any fusion reaction can also be calculated easily, even without conducting the reaction. This has been the real contribution of Einstein's mass-energy equation in the field of atomic energy.

### ***In Search of a Job***

*If schooling was insipid, and college examinations a considerable obstacle, job-hunting proved to be rather difficult for Einstein. He graduated from the Zurich Polytechnic in the year 1900. Most of his friends got jobs in the Polytechnic itself or in Zurich University, but he had to struggle a lot. This was probably because some of his professors were not happy with him. He was hired by a boarding school to give after-school coaching to students, but was fired soon because he had his own way of conducting his classes. Like this, he tried several small-time jobs for a month or two each. He did not mind doing any kind of work; yet he remained more or less unemployed for two years. His father's business continued to be in a bad shape. Still, Einstein was not worried or depressed, such was his inner contentment. Finally, his friend Marcel Grossman, whose father's friend was the Director of the Patent Office at Berne, came to his rescue. Thereafter, Albert Einstein was appointed a "Technical Person of Third Grade" in the Patent Office at Berne. It was while working here that he did several epoch-making researches including the one on the Special Theory of Relativity.*



## Five

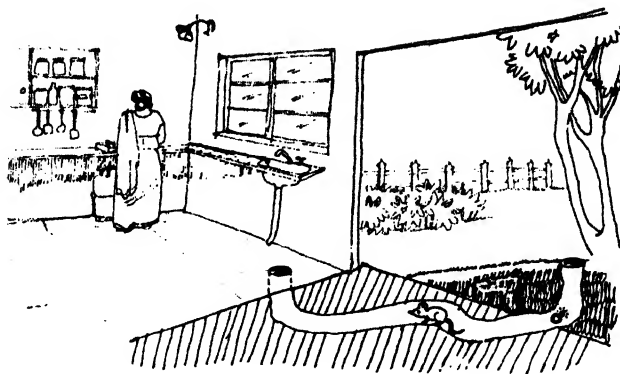
***Romping in Four-Dimensional Space-Time***

*In this chapter, the four-dimensional development of the Special Theory of Relativity is presented in a simple way. One-, two- and three-dimensional spaces, flat and curved, are discussed to prepare the mind to visualize four dimensions. Time is introduced as the fourth dimension. It is explained how its character is similar to that of space and how it is not. Space-time graphs are introduced and the physical meaning of a rotation of space-time axes explained. Finally, the concepts of four-dimensional 'interval', 'world-lines' and 'light-cones' are taken up.*

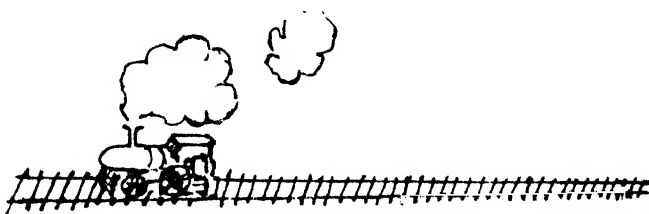
**1. How Many Dimensions?**

How many dimensions are there in the space in which we live? If the meaning of 'dimension' is not clear to you, the question may be put in this form: how many different (independent) directions are there? You might say, "Six directions – East, West, North, South, Up and Down". This has been realized for a long long time. Even Vedic *mantras* are there to meditate on God 'present in all six directions'. Mathematically, however, we say that three directions or axes are sufficient – you may call them East, North and Up. Distances along the other three directions may be denoted by negative numbers. For example, 4 km West may be described as – 4 km East. Thus, our space is three-dimensional. The position of any point in the town with reference to a central crossing may be described by three numbers – so many km East, so many km North and so many m or km Up. The size of a room too is known by three numbers like 15', 12', 10'. This means the room is 15 feet long, 12 feet wide and 10 feet high.

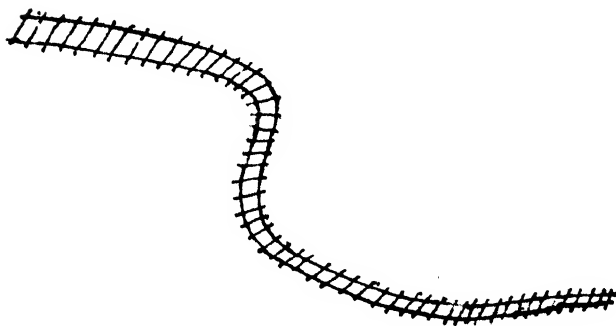
It is not necessary for *every* space to be three-dimensional. There are spaces with one and two dimensions too. Besides, spaces can be flat or curved in many ways, as will be explained below. As far as spaces with less than three dimensions are concerned, these are visualized and understood very easily. As an example, you may imagine a tunnel dug by rats from some point in your kitchen to the vegetable garden outside (Fig. 27a). The tunnel is ver



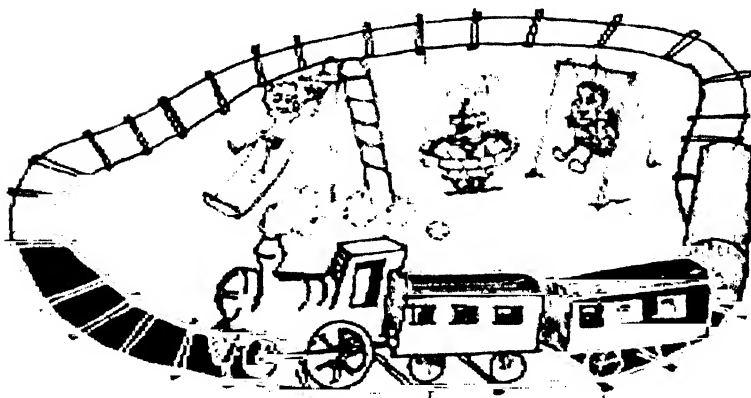
(a)



(b)



(c)



(d)

**Fig. 27 : Examples of simple one-dimensional spaces : (a) rat's tunnel; (b) railway track – straight; (c) railway track – curved; (d) railway track – closed upon itself**

narrow; so its width and height may be neglected. Suppose Mr. Rat picks up a ring from the house and puts it somewhere in the tunnel. Now, to specify the position of the ring, we need only *one* number: the distance (along the tunnel) of the ring from the mouth of the tunnel. Therefore, the tunnel is a *one-dimensional* space, which may be straight or curved.

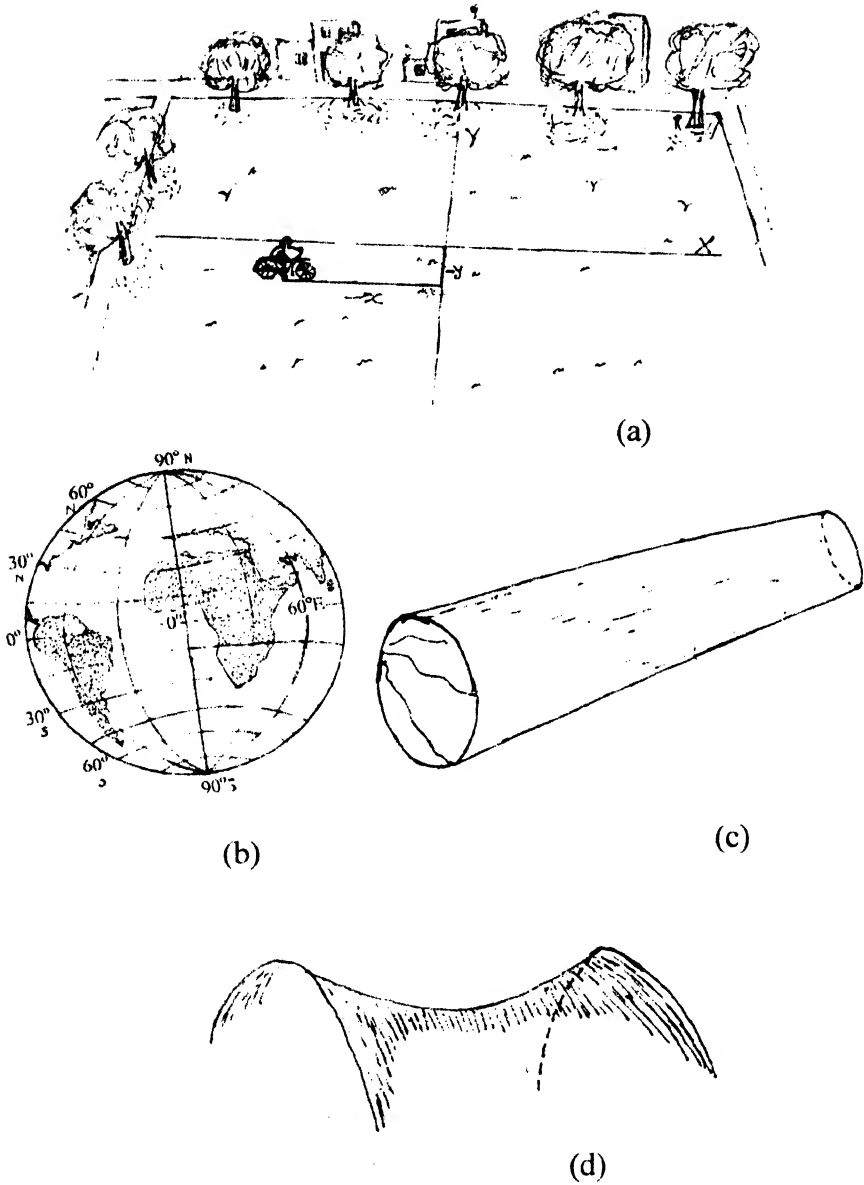
If you don't like rats, you may think of a railway track without any branches or junctions. This, too, is a one-dimensional space. Obviously, such a one-dimensional space may be straight or curved (Fig. 27b,c). It may even curve so much as to close upon itself — see Fig. 27(d). If a rail engine goes on and on 'straight' along such a track, it will reach its starting point after covering a certain distance. Such railway tracks can be seen in the Nicco Park of Calcutta, Appu Ghar of Delhi and other amusement parks.

While an engine on a junction-less track moves in one dimension, a car or bicycle in an open ground, or a ship on the sea, moves in *two dimensions* (Fig. 28a) because *two* numbers are required to specify its position. Note that two-dimensional spaces, too, may be of the flat or curved varieties. The surface of our Earth is a curved two-dimensional space closed upon itself (Fig. 28b). We need two numbers to specify the location of any city on this surface. These may be two angles: for example, Lucknow's position is given by its latitude ( $27^{\circ}\text{N}$ ) and longitude ( $81^{\circ}\text{E}$ ). They may also be two distances: Lucknow is situated at -200 km North and 278 km East with reference to New Delhi.

As mentioned above, the Earth's surface is a curved two-dimensional space *closed upon itself*. The shortest distance between two points in this space is not unlimited: it cannot be more than half the circumference of the Earth's equator. If the Earth's surface were flat (as was believed in olden days), it would be an example of a flat space; such a space cannot be closed (Fig. 28a). There can be a space that is closed in one direction but open in another, as the surface of a cylinder (Fig. 28c). There also exist open curved spaces like a horse-saddle (Fig. 28d) which curves 'inwards' in one direction and 'outwards' in another.

The spaces described above are the simplest examples of their respective types. In actual life we come across much more complicated spaces too. What would you call the network of railway tracks scanning our whole country? It is a set of one-dimensional spaces, with branching and sub-branching, spread on a two-dimensional curved surface (our Earth's surface).

Now, what about three-dimensional spaces? We are already well familiar with three dimensions. All we need to know is whether our three-dimensional space is flat or curved. Could it be so much curved as to be closed upon itself? That is, if we ride a space-ship and go on moving, ever so fast, ever so straight, piercing through the stars and galaxies, shall we come back in the end to the starting point? This question is hard to answer. If you look closely at the above examples, you will realize that living within our three-dimensional world, it is not easy to look for its curvature. Look, it was easy



**Fig.28: Examples of simple two-dimensional spaces : (a) flat; (b) curved and closed upon itself; (c) curved and semi-closed; (d) curved and open**

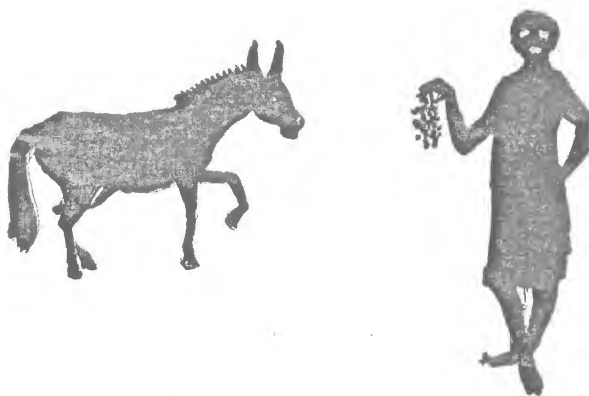
to 'see' the curvature of *one-dimensional* spaces shown in Figs. 27(c) and (d) because the picture was made on a *two-dimensional paper*. Confined to one dimension, we might never have found it out. Likewise, the curved nature of a two-dimensional space, say, the Earth's surface, cannot be shown well in a picture like Fig. 28(b) drawn on a two-dimensional paper. What we need for this is a *globe*, which is a *three-dimensional* object. Extending this

argument, we can say that finding the curvature of our three-dimensional space would be easier *if we could picture it embedded in a four-dimensional space*.

Well, do four-dimensional spaces exist? If so, how do these look? This question appears, if anything, harder than the question of curvature. Just what could be the meaning of a fourth dimension? Is it possible for us to move in a dimension different from East-West, North-South and Up-Down? Perhaps someone will say, "Yes, sir, we can move in the North-East direction too which is called *Ishan*". Likewise, other intermediate directions named *Agneya*, *Nairtya* and *Vayavya* are mentioned in Indian literature. So, do these directions represent additional dimensions? No, because by moving in the North-East direction, you cannot get to any point which cannot be reached by moving northwards and eastwards. So, North-East is not an independent dimension.

If the question was one of merely assigning a fourth co-ordinate to a point, any physical quantity could serve the purpose. Suppose we call the atmospheric temperature as the fourth dimension. Now, there will be four co-ordinates of any point: eastward, northward and upward distances (from the given origin), *and the temperature* at that point. So, does it make a four-dimensional space? Certainly not. The reason is that it is essential for the four kinds of distances to be *convertible into each other* if the space is to be called four-dimensional. In Chapter Two, we saw that, in a two-dimensional picture, if the directional axes were rotated a little, 'northward' distances could get partially converted into 'westward' ones. So, here we must ask: can there be some kind of rotation of axes which would turn a 'northward distance' into a 'temperature difference'? If not, we will have to admit that temperature is not of a nature similar to the space co-ordinates; it is entirely different. Therefore, such a contrived set of four co-ordinates does not represent a four-dimensional space.

The well-known science writer George Gamow has talked, in this context, about two-dimensional shadow-creatures of a two-dimensional world. You must have seen shadows of objects and animals in the light of the Sun. As an animal moves, its shadow also moves. Dr. Gamow has imagined the shadows themselves to be living beings. These shadow-



*Fig.29: Two-dimensional shadow-world in which the man cannot see nor eat, while the 'right-looking' donkey cannot move leftward*

creatures can move only on the two-dimensional surface of the Earth and can see and hear in only two dimensions. Such two-dimensional creatures have no idea at all about a third dimension or a three-dimensional world. If you too forget the world of three dimensions for a few moments and consider this shadow-world as real, you will find it very strange. For example, you will see that the shadow-man in Fig. 29 does have a face, but there is no way anything can reach his mouth. So, you will think, the poor fellow cannot eat the grapes he holds in his hand. He cannot see in spite of his beautiful and good eyes because these are surrounded by body tissues on all sides. Then there is a donkey – in fact, only a side-profile of a donkey. He can eat grapes and see with one eye. He can walk and see only in one direction (to the right). The only way he can move to the left is by backing up – because a ‘right-looking’ donkey can never become ‘left-looking’ while remaining on that surface. (If you try to reverse the donkey’s direction by rotating it on the surface, you will find the poor donkey inverted, with his legs kicking up in the air!)

However, actual shadows of three-dimensional beings do not behave that way. A donkey’s shadow can turn and a man’s shadow can eat. You may be puzzled how it can happen. The only way to solve the puzzle is to recall that *shadows are not complete in their two-dimensional forms, but are only projections of three-dimensional creatures or objects.*

Now, just think about this. Could it not be that we humans and all other living beings and objects of our world, supposedly three-dimensional, were actually just *three-dimensional shadows of some four-dimensional ‘super-objects’*?

Many of our sages have been telling us all the while that what we call the world is not real – it is only a shadow of the Supreme Reality. According to them, all paradoxes of this shadow-world stand resolved when one knows that Supreme Reality, just as all paradoxes regarding the two-dimensional shadow-world were resolved above by knowing the three-dimensional reality. But let us not go into this metaphysical discourse. *In physics*, how can we know a four-dimensional reality of which we may be shadows? What is it and how is it related to our familiar three dimensions?

## 2. Four-Dimensional Space -Time

With the theory of relativity, the riddle of the fourth dimension has been answered. This theory, as developed by Minkowski and others, says the fourth dimension is *Time*. The real things in nature are not points or objects in space, but happenings or ‘events’. An event can be described as a point in the four-dimensional space-time. It has not only three space-co-ordinates associated with it, but also a time co-ordinate. For example, if we describe the Fifth National Children’s Science Congress, we specify not only the venue, viz. Bhopal (latitude 23°N, longitude 77°E, height 400 m above mean sea-level), but also the *time* when it occurred. Just as a body has spatial dimensions like length, breadth and height (or thickness), it also has a *time-interval*

associated with it. For example, a house is 17 metres long, 10 metres wide and 7 metres high. If the house was constructed in the year 1915 and destroyed in 1970, this 55-year interval will be the time-dimension of the house or its 'thickness' in time.

You may find it somewhat difficult to imagine Time as an independent direction in addition to East, North and Up. In the last section, we saw that temperature could not be considered as a fourth dimension as temperature-differences could not be converted into distances. Perhaps you may raise the same objection here: time-intervals too cannot be converted into distances. But Einstein would say, "Well, you have got the wrong impression that distances and time-intervals are very different kinds of quantities because your experience is so severely limited. If you had traveled at speeds like one or two lakh km per second, or at least studied objects moving that fast, you would have readily realized that, just as the space co-ordinates  $x$ ,  $y$ ,  $z$  can be converted into one another, so can they into time".

In Chapter One, it was seen that if, on a moving train, two events occur *at the same place at two different times*, then an observer standing on the ground will see them occurring at *different places*. The theory of relativity says that if only the speed of the train were sufficiently large, the opposite truth would also become equally apparent: if two events take place on the running train *at the same time but at different points*, the observers standing on the Earth will see them happening *at different times*! Indeed, spatial distances can be converted into time-intervals, and the opposite is also true.

You might say, "But how is that possible? Distances in the eastward, northward and upward directions are measured in metres. Can time be measured with a metre-rod? Or, can eastward or northward distances be measured in minutes or seconds?" This objection can be dealt with rather easily. If you are asked how far your office is from your house, you might just say, "About twenty minutes"! What you mean to say is that it takes twenty minutes to get there *by walk*. Since walking speed is about 6 km per hour, your 'twenty minutes' are equivalent to 2 km. If this time had been specified with a car-ride at 45 km per hour in mind, 'twenty minutes' would have meant 15 km. So, it is clear that distances can be measured in time-units, provided a standard speed is agreed upon. Now, is there a universal standard of speed which is the same under all circumstances? In Chapter Two, we saw that the speed of light in empty space, which is also the maximum speed of any physical interaction, is quite absolute. Therefore, taking this speed as standard, distances and time-intervals can be inter-converted. Indeed, you must have actually heard or read very large distances being quoted in 'years' or 'light-years'. Thus, our nearest star (other than the Sun), called Alpha Centauri, is said to be 4 light-years away. Distant galaxies seen faintly with the help of the largest telescopes are estimated to be billions of light-years away.

It may be noted that expressing spatial distances in time-units becomes

practically useful only when the distances involved are astronomically large. If an 'ordinary' distance (e.g. 1 metre) is to be expressed in time-units, it would come out to be an extremely small fraction of a second. Likewise, if an 'ordinary' time-interval (say, several years) were to be written down in metres, the digits would fill up a whole line, and perhaps no language of the world would have words to describe such large numbers!

So, it would appear that time is equivalent to space, just like energy was shown to be equivalent to mass, with a constant factor for conversion. But wait! There is another objection. Along the East-West axis, one may move westwards just as easily as eastwards. The same can be said about the north-south axis. Birds flying in the air can climb down as well as up. But *can we go backwards in Time?* Can we reach 'yesterday' again? Is it possible to see in the real world the kind of sequence seen in a motion-picture run backwards: digested food being 'chewed out' of the mouth and being converted into whole potatoes, tomatoes, cauliflowers and grains of *arhar dal* and rice, which in turn get planted into the fields....? In such a time-reversed world, if an old man were asked about his parents' names, he might blush and say, "Well, how can I foretell who my parents are to be. It has been just a few years since I rose from the cremation grounds and am still trying to find who my children are". If all of this appears bizarre and impossible, we shall have to admit that time is not *exactly* like dimensions of space, though it may be a part of the four-dimensional space-time.

Now, let us show how distances in space may be converted into time-intervals and vice-versa. To this end, it will be necessary to draw a diagram showing space and time dimensions. But there is a problem in showing all four dimensions. Our paper being a two-dimensional plane, we can draw only two dimensions on it satisfactorily. Therefore, we draw a graph with only one dimension (x) of space, besides the time (t) (Fig.30). In this figure, the horizontal x-axis represents distances along the Delhi-Allahabad railway line. A particular tea-stall at Kanpur Central Station is considered the origin, from which distances are measured. The time is plotted on the vertical axis. Consider two imaginary events – point A represents a collision between two rail engines, which occurs at a distance of +20 km at 8.00 a.m. The other event is the inauguration of a new railway over-bridge by the Railway Minister the same day at 11.00 a.m. at +100 km, represented by point B. Clearly, the spatial distance between the two events is  $AC = 80$  km. And the time-interval is  $BC = 3$  hours.\*

Now, if we rotate the cross made by these axes by some angle, what will happen? In Fig. 30 we have rotated the axes such that the angle between the new space and time axes is still a right angle. In this new system, the 'spatial' distance between the two events is not AC but  $AC'$ , and the 'time-interval' has also changed to  $BC'$ .

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\*It must be remembered that, in such space-time graphs, space and time must be expressed in the same unit, say km. Graphs like that shown in Fig. 30 will be meaningful only when the spatial distances involved are huge.



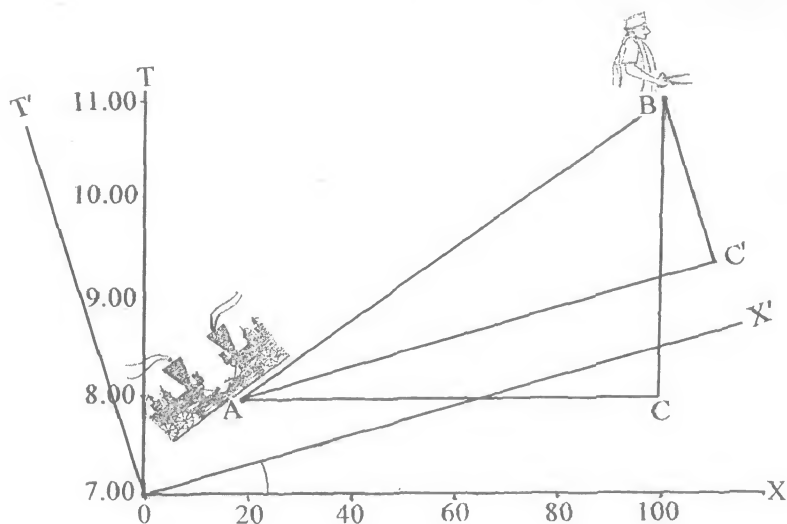


Fig. 30. A time-distance graph (The scales on both axes are not similar)

According to the theory of relativity, measurements made from a moving space-ship or laboratory are like measurements made with reference to a set of rotated  $x$ - $t$  axes as shown above. If the first set of axes corresponds to a laboratory fixed on the Earth, then the rotated axes will show spatial distances and time-intervals as measured from a space-ship flying at a speed  $v$ . The angle  $\theta$  at which the axes will have to be rotated is such that  $\tan \theta = v/c$ . It is also necessary to keep in mind that time is somewhat different in nature from spatial distances. As a consequence, the vertical axis in Fig. 30 represents not quite the time  $t$ , not even  $ct$ , but  $ict$ , where,  $i = \sqrt{-1}$  is the unit of imaginary quantities.

It is possible to calculate contraction of lengths or dilation of time-intervals on the basis of this rotation of axes, but we shall not take it up because it will be necessary to deal with imaginary quantities, with which readers may not be familiar. But it must already be clear that a rotation of the space-time axes may bring about changes in lengths and time-intervals, and convert one of these into another. (Remember how westward and northward distances got mixed up in Fig. 7, Chapter Two?) Making measurements from a moving rocket has the same effect. It is also clear that, where the velocity  $v$  is much smaller than  $c$ , the velocity of light,  $v/c$  will be very small; therefore, the angle  $\theta$  will also be negligible. Under such conditions, the space-time axes will undergo almost no rotation, nor will there be any change in lengths or time-intervals, which is what we see in daily life.

In Chapter Two, we had discussed the distance between two points

(house and office). We had found that a rotation of the co-ordinate axes could change the projections of this distance on the East and North axes, *but the straight distance  $L$  did not change*; this distance was the square root of the sum of squares of both projections:

$$L = \sqrt{L_1^2 + L_2^2}.$$

Likewise, in three dimensions, we shall say the straight distance  $L = \sqrt{L_1^2 + L_2^2 + L_3^2}$  does not change upon a rotation of axes. Now, it is natural to ask if there is a four-dimensional 'interval' which is the same in all laboratories, i.e. which does not change upon a rotation of the space-time axes, even though lengths contract and time-intervals expand. The answer, according to the theory of relativity, is that if one event occurs at point  $(x_1, y_1, z_1)$  and time  $t_1$  and another one at  $(x_2, y_2, z_2)$  and time  $t_2$ , then the *four-dimensional interval*,

$$P = \sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2 (t_2 - t_1)^2\}},$$

will be such an *invariant* (unchangeable) quantity. It may be noted that here the squares of distances along various spatial axes have been added together (in accordance with the Pythagoras' Theorem), but *the square of the time-interval has been subtracted*. The reason is that, as explained above, time has to be multiplied by the imaginary quantity  $\sqrt{-1}$ , whose square is  $(-1)$ . Thus,

the value of the 'interval'  $P$  between two events will have the same value in all inertial laboratories\*, although their spatial distances and time-intervals will have different values. We may say that the three-dimensional spatial distance  $L$  is just a projection or a shadow of this interval  $P$  on the spatial axis. Similarly, the time-interval  $T$  is its projection or shadow on the time-axis. Thus, the famous teaching of the *Vedanta* philosophy, "*Brahma satyam jagat mithya*", may be re-stated in space-time physics as "Four-dimensional interval is *satyam* (real or absolute), while spatial distances and time-intervals are

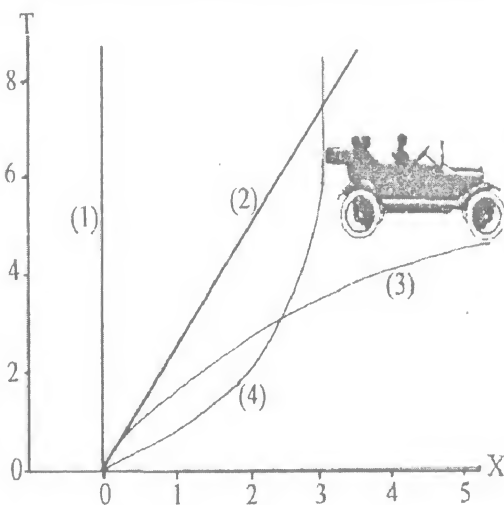


Fig.31: (a) World-lines of a car : (1) stationary; (2) constant velocity; (3) increasing velocity; (4) decreasing velocity

is *satyam* (real or absolute), while spatial distances and time-intervals are

\* 'Inertial' means at rest or in uniform motion, i.e., not accelerating.

*mithya* (changeable or relative to the state of motion of the reference laboratory)”).

### 3. World-Lines

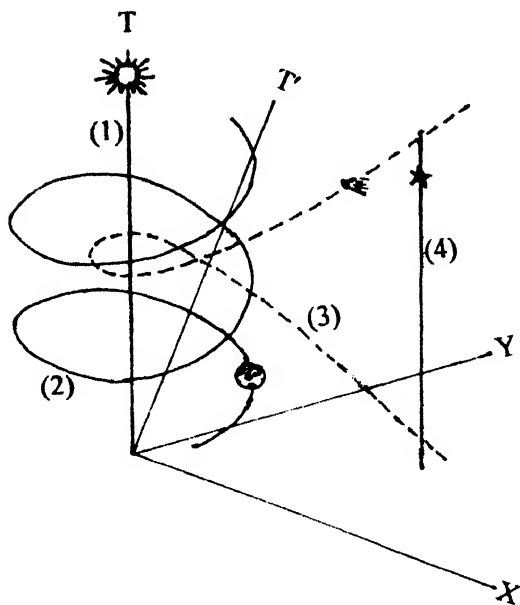
Next, we consider the motion of a body and the force acting on it. In this, we shall find very helpful a graph in which one axis represents time and the other axis or axes the spatial co-ordinate(s), i.e. distance(s) from a particular point. In such a graph, the motion of a body can be described by a line. Besides other things, one can see clearly from this picture whether a force is acting on the body.

As an example, in Fig. 31(a), we plot one-dimensional distance ( $x$ ) along the horizontal axis and time ( $t$ ) along the vertical one. This graph shows four different kinds of motion of a car. Line (1) shows the car at the 0 km position at all times. Therefore, it is stationary. Line (2) too is a straight line: the car travels one km every 2.5 minutes. Therefore, the car is moving at a steady velocity of 24 km per hour. In case (3), the car starts out as in the previous case, but its velocity goes on increasing with time. Line (4) shows that the car starts faster than in case (2), but its velocity goes on decreasing and, in the end, it stops at the 3 km mark. Note that the *curvature* in lines (3) and (4) signifies *changing velocity* of the car, which indicates that *a force is acting on it*.

Such a line, which shows the motion of a particle or body in the graph of space and time, is called the *world-line* of the body. The above discussion makes it clear that when there is no net force acting on a body, i.e. when it is moving with a constant velocity, its world-line is a straight line. Curvature in the world-line means a change in the magnitude or direction of its velocity.

Note that the world-line being curved is one thing and a curved path of the body in two- or three-dimensional space is a different thing. Of course, if the path of a body in space is curved, then the direction of its velocity must be changing; so its world-line must also be curved. But the reverse is not true. In Fig. 31 (a), world-lines (3) and (4) are curved while the path of the car is quite straight. The path of a rain-drop falling down is also straight, although its world-line must be curved as its speed increases with time.

Let us now see how the world-lines of members of our solar system look. Fig. 31(b) is a three-dimensional graph in which two spatial axes ( $x$  and  $y$ ) are represented along with the time axis, which makes a right angle with both. Here, line (1) is the world-line of the Sun. Since the Sun has been assumed nearly stationary, its world-line is a straight line parallel to the time-axis. The Earth goes round the Sun in a nearly circular orbit; so its world-line (2) traces out a helical (spring-like) curve as time goes. If we consider a comet which approaches the Sun from far away, comes very close and again goes away, we get a world-line like that shown in (3). If other stars too are considered fixed, each of them (like Alpha Centauri) will have its world-line going straight upwards like (4) in the Fig.



**Fig.31: (b) World-lines of some celestial bodies: (1) the Sun; (2) the Earth; (3) a comet; (4) another fixed star**

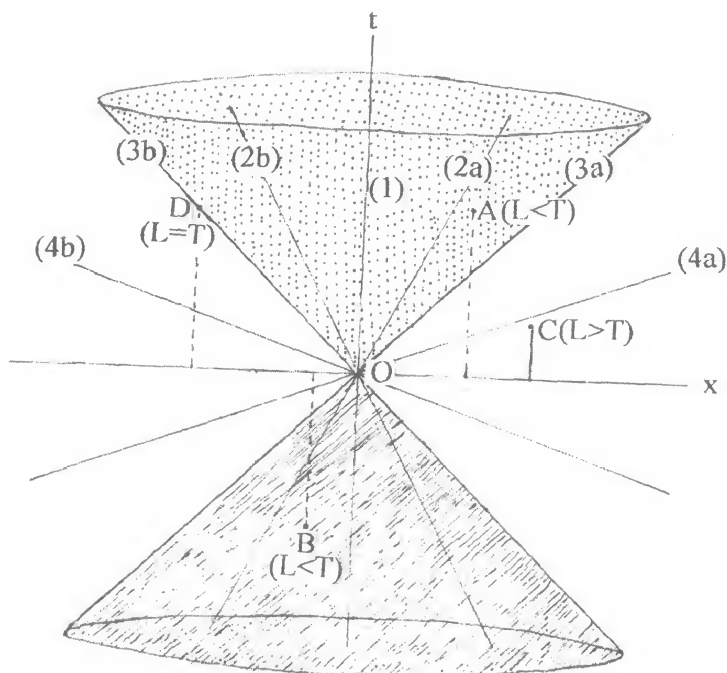
Now, if these bodies were seen from a space-ship-moving at a constant velocity relative to the Sun, this would be equivalent to a rotation of the space-time axes. The world-lines of the Sun and other stars (1, 4) would remain straight lines, though making a certain angle with the time-axis.

It has been pointed out in Chapter Three that no object or particle can move with a speed greater than  $c$ , the speed of light. This divides space-time into regions accessible and inaccessible from a given point. To show this, we plot spatial distance ( $x$ ) and time ( $t$ ) along the two axes

in Fig. 32. Note that, here, time has been expressed in units of length (metres, i.e. light-metres), so that both distance and time may be plotted on the same scale. Now, suppose an object is at the origin  $O$  at time  $t = 0$ . If its speed is zero, it will remain at the same point in space, and its world-line will be along the vertical axis (1). If its velocity is half of the velocity of light, its world-line will be (2a) or (2c), which make an angle of nearly  $27^\circ$  with the time-axis, because on such a line the object would move one metre in a time of two light-metres. Line (2a) shows the object moving in the positive direction and line (2b) in the negative direction. Lines (3a) and (3b) make an angle  $45^\circ$  with the time-axis. Here,  $\tan \theta = 1$  and  $v = c$ . Motion along such a line implies a velocity equal to that of light; therefore, these lines can represent world-lines of light only. Now we come to lines (4a) and (4b), which show velocities greater than that of light. Clearly, such lines cannot represent the world-line of any object. Thus, *only the region of space-time between lines (3a) and (3b) is available for the world-line of any object passing through  $O$* ; the outer region is not allowed. Now, if we include two dimensions of space ( $x, y$ ) in the picture rather than just one, the available region will take the shape of a cone. The vertex of this cone lies at the origin  $O$ , its axis is the time-axis and half-angle is  $45^\circ$ .

The above discussion was about the future ( $t > 0$ ). We may also ask questions about the object's motion in the past, i.e. we can try to draw its world-line for  $t < 0$  too. The reader may see easily that the world-line of this object in the past must also be somewhere within the bottom cone in Fig.

32. So, altogether, we may say that any world-line passing through a point O has to lie wholly within the two cones with that point O at the apex, not



*Fig.32: The light-cones : any world-line passing through the origin O must have a future within the upper cone and past within the lower cone, never outside.*

outside, i.e. not in the shaded region. These two cones are called *light-cones*. These light-cones can be drawn at any given point in space-time.

If you remember that  $c$  is the upper limit of the speed not only of a particle, but also of any interaction, you will realize that the light-cones shown in Fig. 32 restrict the possible effect of one event on another too. For example, the event O can only affect events like A, which lie within its future light-cone. The event O itself must be caused by events within its past light-cone (like B). It can have no cause-effect relationship at all with events like C, which lies outside its light-cones. *These results are true absolutely, i.e. irrespective of the motion of the laboratory from which the events are viewed.*

To understand why the last statement is true, it may be stated in terms of the four-dimensional interval  $P$  between two events defined in the previous section:

$$P = \sqrt{(L^2 - T^2)},$$

where  $L$  is the spatial distance between the events and  $T$  their separation in time, measured in length units. If  $L > T$ , the separation  $P$  is real, and it is called *space-like*. It is possible in this case to find a moving laboratory in which the two events will be separated only in space, not in time, i.e.  $T' = 0$ , or the events are simultaneous. Obviously, such events, like  $O$  and  $C$ , cannot be related causally. On the other hand, if  $L < T$ , the separation  $P$ , being imaginary, is called *time-like* and may be seen as a pure time-separation in some suitable laboratory. These two events may have a cause-effect relationship, e.g.,  $O$  can be the cause and  $A$  its effect, or  $B$  can be the cause and  $O$  its effect. The time-sequence of these two events cannot be inverted. If  $L = T$ , the four-dimensional separation is zero. This does not necessarily mean that the two events occur at the same place and same time, but only that these may be connected by a light ray, like  $O$  and  $D$ .

We shall again discuss these world-lines in the next chapter, where Einstein's General Theory of Relativity will be considered.



### *Who Is the Third Expert?*

*The various concepts occurring in Einstein's Theory of Relativity were so far from common-sense notions that even scientists found it hard to visualize these. Once the famous British astronomer Arthur Eddington (who was the first to come up with an experimental verification of the General Theory) was told by a colleague, "Professor Eddington, I suppose there are just three scientists in the whole world who understand the theory of relativity, and you are one of them." Eddington did not say anything; he looked surprised. The colleague then added, "Don't you believe it, Professor? You are just being modest ..."* Eddington said, "Leave alone modesty. I was just wondering who the third fellow could be!"

## Six

# *Gravitation and the General Theory of Relativity*

*Situations are visualized in which loss of weight or weightlessness may be experienced. Resemblance of fields of acceleration with the field of the Earth's gravity are also discussed. Einstein's Principle of Equivalence is presented as a generalization of these experiences. It is explained how the curved world-line of an object in a gravitational field may be seen as the straightest possible line (geodesic) in a curved space-time. The possibility of obtaining a truly inertial reference-frame is presented, as also the fact that it is only locally inertial. Characteristic results of the General Theory of Relativity like precession of Mercury's orbit, dilation of time, bending of light rays in gravitational fields and the possibility of black holes are explained.*

### **1. Getting Rid of Weight**

If only there were an easy method of reducing weight, fat people would get a great relief. So, let us see how weight may be reduced. But, remember, here we are discussing physics; therefore, whatever method of reducing weight is suggested here will be applicable as much to a stone or a statue as to bodies of men, women and horses.

You must have used a lift (elevator) in a multi-storey building or taken a joy-ride in a swing climbing up and down rapidly. Have you noticed how the body-weight is affected in such a situation? As the lift begins to climb up, one feels as if one's weight had increased, as if one were being pressed downward. Once the lift gets into uniform motion, everything becomes normal. But as it begins to slow down, once again there is an abnormal thing – a feeling of being much lighter if not floating in the air.

This feeling of increase and decrease in weight is not an illusion. You can measure it if you wish. All you have to do is to stand on an ordinary weighing machine (bathroom scale) in the lift. As the lift begins to rise, the machine indicates an increased weight and, when it is slowing down, the weight indicated is less than your normal weight (Fig. 33 a). While climbing down, the opposite of the above sequence occurs: you are under-weight near the beginning and



(a)



**Fig.33:** The changes in weight of a passenger inside a lift : (a) while ascending; (b) while descending

over-weight near the end (Fig. 33 b).

What about getting rid of weight entirely? You know that weight is felt due to gravitation. The Earth applies a force of gravity on each particle of our body (as indeed on every other object), i.e. pulls it down towards its centre; this force is what we call 'weight'. Now if gravity itself could be abolished somehow, the weight would automatically vanish. The hefty Seth Kaddulal, the lean Master Lakir Chand, a small insect – all will be equally weightless. It would feel wonderful, isn't it? But the million-rupee question is: *how can gravity be abolished?*

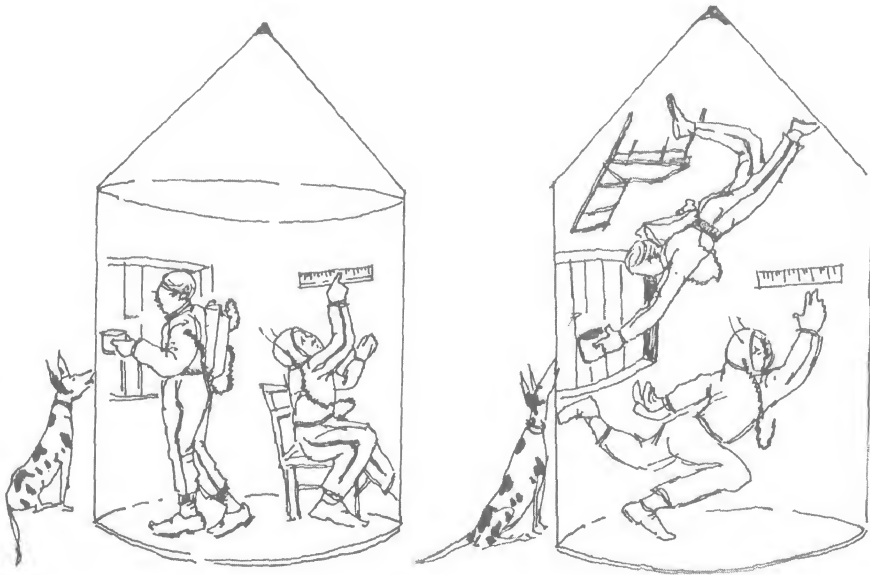
Sometimes, in science-fiction stories, a situation is presented in which there is a secret room in a house where there is no gravity. Is such a thing really possible? Not at all. Gravitational attraction is not like electrical attraction where the effect of positive charges can be neutralized by negative charges. In gravitation, there is nothing analogous to negative charges. Wherever you go on the surface of the Earth, gravity will be there. There can be no shielding. And, of course, you cannot place another earth on the other side of the room to offset the Earth's gravity. (Where would you get the other earth?)

One possibility is that you ride a space-ship and go into the outer space – to a location very far from the Earth, the Sun and all other massive bodies. In such a situation, you will feel quite weightless. Not only you but all living and non-living bodies around will be able to float freely in all directions. Now, this may be all right for the likes of Rakesh Sharma and Kalpana Chawla, but can there be no hope for ordinary mortals? Surely hope is there, and the method involved is so very simple that you will be surprised. The only problem is that the weightlessness can be enjoyed only for a fraction of a second at a time. All



right, what you have to do is to come out into the open and take a jump. You may also try jumping from a height of a metre or so (carefully!). It does not matter whether it is a 'long' jump or a 'high' jump, whether you are going up or down. *As long as you stay up in the air, you will be almost entirely weightless.* (It is a different matter that you may forget all the joy of weightless floating as soon as you fall down with a thud!) Such a situation is called a *free fall*. Any satellite, whether a natural one like the Moon, or a man-made one like INSAT 1-D, will be considered as falling freely as long as it is not being driven by an engine. All persons and objects moving with it will also be weightless.

The famous French novelist Jules Verne, in his novel, "A Journey around the Moon", written two hundred years ago, has portrayed an engineless space-craft being propelled into space by a huge launcher gun. The space-craft rushes away from the Earth together with several men, pets and other required materials. A dog called Satellite dies from the shock of the propulsion; he is dropped out of the window. Later, the astronauts are amused to find that the dead body of



*Fig.34: The inside of an engineless space-craft: (a) as imagined by Jules Verne (wrong); (b) the correct view (weightlessness)*

Satellite has not fallen to the Earth, but is moving beside the space-craft. Jules Verne probably reasoned that whatever effect the Earth's gravity had on the space-craft, the same effect must be there on the dog too; therefore, both would remain side by side. This line of thinking is entirely correct. But he made an error in showing the astronauts standing on the ground inside the space-craft just like they would on the Earth (Fig. 34a). The true situation is that not only the dog and the space-craft will be in a free fall, but also the

astronauts and all other things inside the space-craft. Therefore, all should be weightless. All should be flying around freely and facing any directions at random (Fig. 34b).

The possibility of being completely weightless may fascinate you, but space-men often see it as a problem. Imagine having a joy-ride in your space-ship into the outer space where there is no gravity. You can switch off all engines and enjoy your weightless state. You may fly around, twist, turn and somersault in your cabin to your heart's content. But soon you get hungry and reach for the table on which you had put a box of sandwiches. Lo! There is no box there. The fact is that the sandwich-box too feels free to fly around, and so do the tools and other things put on the table. Indeed, the table itself would be flying around, had it not been fixed rigidly on the floor. With some effort, you catch hold of the truant sandwich-box and open it. You must hold on to the sandwiches, lest they should fly away in the slight breeze of the fan. And now you eat your sandwich. But what if it refuses to go 'down' into your stomach? Feeling thirsty, you may not drink water from an open glass, nor

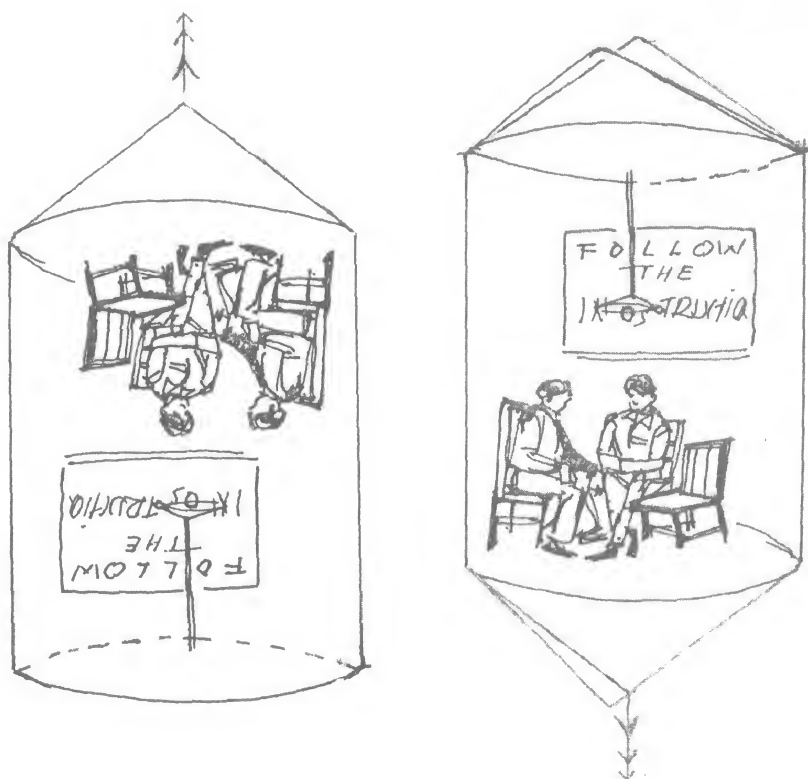


Fig.35: Producing 'gravity' artificially: (a) while speeding up; (b) while speeding down

may you ‘pour’ water from the jug into the glass. You have to suck it through a pipe. And the water, once inside your body, feels as free to go ‘down’ as to move ‘up’. In fact, you find it hard even to settle down in your chair or stand in a steady manner.

So, now, we are forced to ask the opposite question: whether some gravity-like force can not be produced artificially. It has been found that increasing or decreasing the speed of the space-ship with an engine can solve the problem. When the speed is decreasing, all the objects will tend to fall to the front end of the space-craft, which will be like ‘ground’ (Fig. 35a). If the speed is increased, then the rear end of the craft will be like ground (Fig. 35b). This is just like what you see in a moving bus: the passengers fall backward when the bus suddenly accelerates and fall forward when the brakes are applied.

## 2. Acceleration and Gravity are Equivalent

In the last few chapters, we have discussed Einstein’s Special Theory of Relativity. We have seen that if one observer is ‘stationary’ and another one is ‘moving’ with a uniform velocity, then various measurements made by both will be equally valid, each in its own way. The length, mass, speed, etc. of any object measured by the two observers will be different from each other; yet each set of measurements will be correct and fully consistent with the basic principles of physics—so much so that it will be impossible to say in an absolute way as to which observer is stationary and which one is moving.

This theory and its results are valid only as long as the relative velocity of the two observers is constant. Not only that, it is also necessary that both observers are ‘inertial’. Broadly, this means that the velocity of each one is constant; there is no acceleration. (No acceleration *relative to what*? There is an implicit assumption here that acceleration is absolute.) The theory is called special because its applicability is limited to these special (inertial) systems of reference.

The question that bothered Einstein was why the applicability of laws of physics must be limited to this special class of systems. Could a more general theory not be formulated which would enable us to apply the same physical laws in accelerated systems too? Ernst Mach (a mathematical thinker) had already put forward the view that acceleration of an object could only be defined relative to other objects and not absolutely. The view appealed greatly to Einstein, but no theory had been constructed to incorporate it.

Einstein was also struck by the fact that, in a gravitational field, every object experiences exactly the same acceleration. For example, take any point near the surface of the Earth. Any object released at this point will fall down with the same acceleration. The value of this rate of change of downward velocity (nearly 10 metres per second per second) is the *same for every object* whether it is made of iron or gold or clay. That is why any two objects dropped simultaneously from a tall tower fall to the ground together (provided the effects of air can be neglected). Not only that, the acceleration due to gravity

is the same irrespective of the direction and speed of motion of the object. Balls rising up, falling down, moving horizontally ..... all undergo the same acceleration. How remarkable!

Just think, if sitting in a train, you see through the window that all outside objects are running with the same speed and in the same direction, what will you conclude? Of course, that your own train may be running in the opposite direction. Similarly, when every object on the surface of the Earth is seen accelerated downwards at the same rate, it is natural for the following thought to arise: is there a downward gravitational field, or are we riding a van whose own velocity is increasing (upwards) at the rate of 10 metres per second every second? The various examples given in the last section too point to the conclusion that the situation in a gravitational field is similar to that in an accelerated system.

Einstein, with his keen insight, realized that this similarity is not accidental, nor is it limited to motions of objects, but is a fundamental feature of nature. Thus, in 1907, he announced the following Principle of Equivalence: *in any limited region, the effect of a gravitational field is exactly the same as of the field arising in an accelerated frame of reference.*

Einstein could see that this principle of equivalence contained the key to a deeper understanding of gravitation. With this, it also became clear that the special theory of relativity was not enough to treat gravitation; a more general theory would be needed which included accelerated systems too. Several centuries ago, Galileo had put forward his principle of inertia, according to which a particle not subject to any net force moves in a straight line with a constant velocity. In the language of space-time, we may say that *the world-line of such a particle is a straight line*. A particle subject to a net force will have a curved world-line. For example, the fact that the Earth's world-line is not straight (Fig. 31b) indicates that a force is acting on it. This force (due to gravity of the Sun) is such that any object placed in the Earth's position will have the same acceleration and same curvature in its world-line as the Earth has. Since every object at a particular place has the same curvature in its world-line, does it not indicate that *the curvature is associated with space-time* rather than with individual objects? Einstein, after many years of thinking, came to the conclusion that the principle of inertia must be generalized in such a way that bends in world-lines arising due to gravitation are included within the 'curvature of space-time', and do not require gravitational forces for their explanation. He tried to find a generalized definition of a 'straight line', which would include the curved world-lines shown in Fig. 31(b) too, as explained below.

### 3. Gravitation and Curved Space-Time

As you know, a straight line may be defined as the shortest path between two points. It will be explained below that 'the shortest path' depends upon the *kind of space* being considered. Suppose, for example, you want to go from

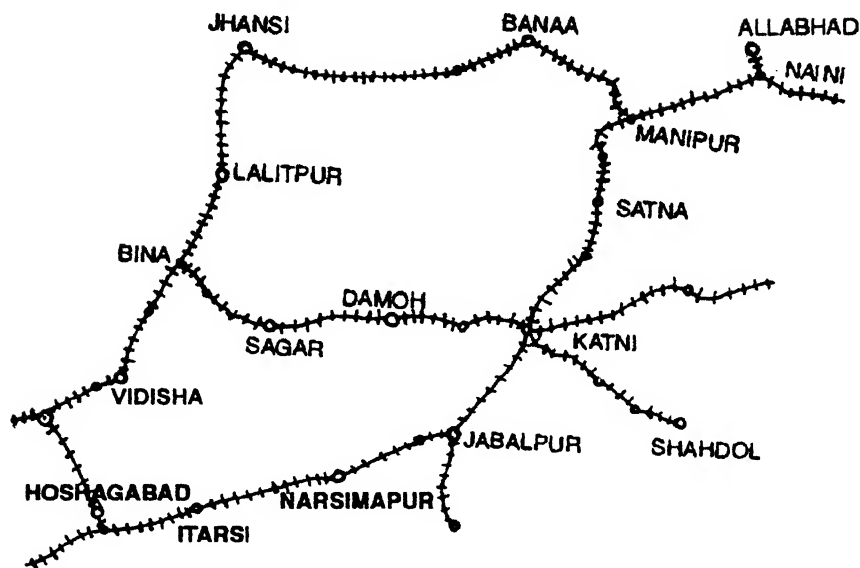


Fig.36: Rail route between Bhopal and Allahabad : is it a straight line?

Bhopal to Allahabad by 'the shortest route'. Will this route be a straight line? Fig. 36 shows the one-dimensional curved space of the railway line in that region. Clearly, no train will travel between Bhopal and Allahabad along a straight line of the kind that you draw with the help of a ruler. Here, 'the shortest route' will mean the shortest path *along the railway line*. Even if you were to abandon the train and travel on foot, just to take the path that was 'really the shortest', you would still not be able to go along a straight line, even assuming no hills, rivers or other obstacles on the way. Why? Because the Earth's surface itself is a *curved two-dimensional space*. (Surely, you are not going to dig a straight tunnel through the Earth!). The best you can do is to walk along a curve which is part of a *great circle*. (A 'great circle' on the surface of a sphere is that circle which is the largest, i.e. whose centre lies at the centre of the sphere.)

By now, it must have become clear that 'the shortest path' between two points, and its length, depend on the kind of space it lies in. If the space itself is curved, it is not possible to move along a straight line. Even so, *there must be some shortest path*: this path is called a 'geodesic' in technical language. It is a generalization of a straight line. You will readily see that, in a flat space, a geodesic (shortest path) is a straight line. On the surface of a sphere (as on our Earth), any arc of a great circle is a geodesic. Likewise, geodesics can be drawn on various other curved surfaces too.

To come back to Einstein's story, he taught that, *in the vicinity of large objects, the space-time itself is curved*. Apart from this, there is no need to talk about any force of gravitation. He generalized the principle of inertia as follows: *the world-line of an object free from all forces (other than gravitational) in*

*four-dimensional space-time is a geodesic.* Where there is no gravitational effect, the space-time will be flat, and the geodesic will be just a straight line. Thus far, it is just a re-statement of the old principle of inertia. But the new element is that, in the presence of gravitational fields, the world-line of an otherwise force-free object is still a geodesic, i.e. the straightest possible line in the curved space-time.

An alert reader might raise an objection here, "Look, the examples of

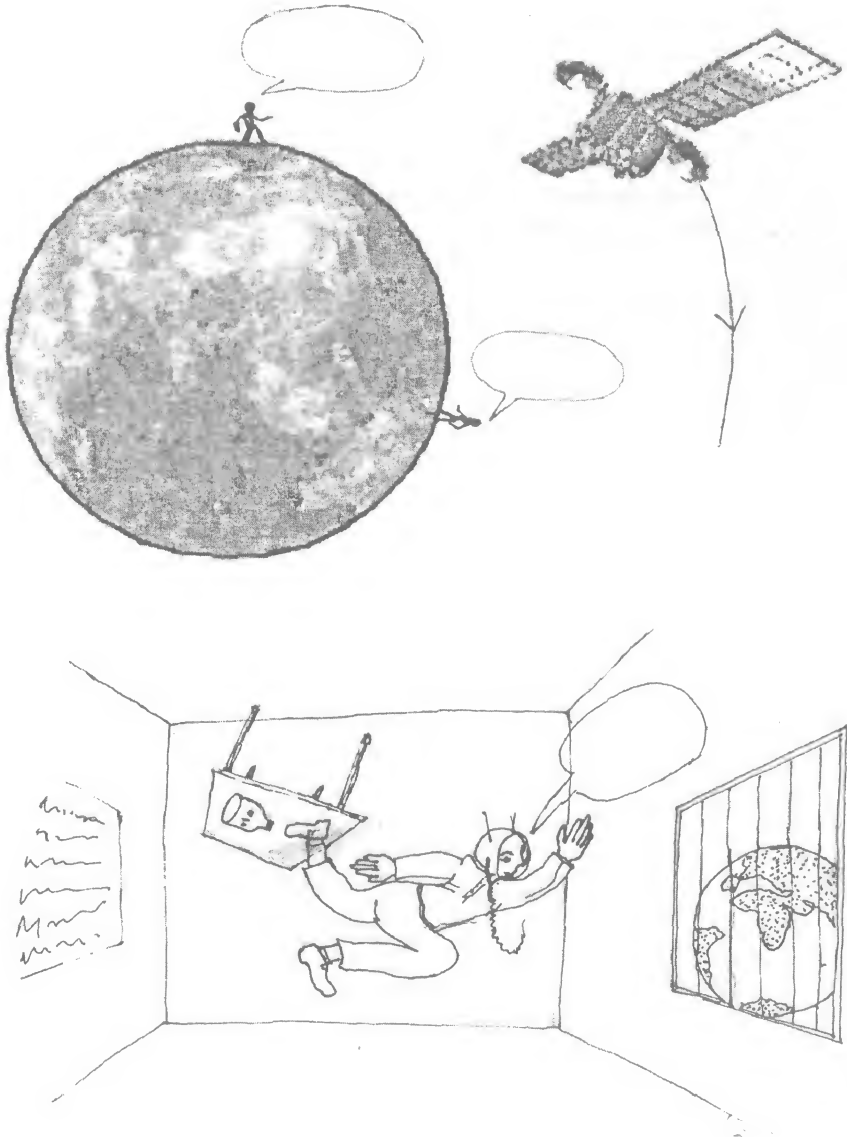


Fig.37: A satellite circling the Earth : (a) as seen from the Earth; (b) as seen from inside

curved spaces you have given are not really convincing. The one-dimensional space of the railway track is curved because the *iron rails* put there are bent into a curve. The two-dimensional surface of the Earth is curved because the *material* of the Earth (rocks, etc.) is assembled in the shape of a sphere. But what can you possibly mean by *empty* space being curved?"

Well, curves in empty space may appear weird and even imponderable. But this is because we have already formed rigid notions about space (and time) even in our childhood. There is no logical reason why space and time should not have *properties* to be studied and matched with simple, beautiful principles like that of all motion being relative. We have already seen in Chapter Two and later chapters that space and time do have a structure, i.e. have properties, which need careful investigation and discussion. As discussed in Chapter Three, an observer moving at a high speed relative to a reference-frame finds all lengths there contracted along the direction of his motion and all clocks there running slowly. If space can shrink, and time can be dilated, it should be no surprise that empty space can bend into a curve in the vicinity of heavy bodies.

To discuss an example of a curved space, consider an engineless satellite circling the Earth (Fig. 37a). Let us ask an observer standing stationary on the Earth as to why the direction of motion of the satellite is changing continuously. If the observer follows Newton, he will say, "Due to the Earth's gravity, of course!" If he takes up the Einsteinian viewpoint, he will say, "No forces. The satellite's world-line is a geodesic, i.e. it is as straight as can be drawn in this curved space-time!" (The circular curve of the path of the satellite in three-dimensional space must not be confused with the curvature of the world-line in four-dimensional space-time. See Chapter 5.)

A most interesting feature of the General Theory of Relativity is that *a particle falling freely under gravity can be taken to be inertial*. In Newton's theory, an inertial particle or an inertial laboratory, i.e. one which is not subject to any acceleration, is only an idealization, never to be actually realized. If you were to explore the entire universe, torch in hand – no, pendulum in hand – you would never find a region of space *completely free* from gravitational forces. After all, the stars and galaxies exert forces even on objects very far off. In practice, however, the astronaut in the satellite in the present example (Fig. 37b) will tell you he is totally without acceleration. And asked about the force of gravity, he says, "What gravity and what force, Sir? There is no force here. Does anything 'fall down' when dropped in my cabin? Of course, not. Look, I am myself floating around in the air!"

We have already discussed such situations of weightlessness earlier in this chapter. It has to do with the Principle of Equivalence, of course. In the present example, too, the Earth-observer sees an acceleration of the satellite towards the Earth's centre (centripetal acceleration). In the accelerated system of the satellite, therefore, there is an *equivalent* gravitational effect in the opposite direction, i.e. away from the Earth. This effect cancels the actual gravitational

effect due to the Earth, leaving the satellite free from all forces and accelerations. A similar analysis applies to any body in a free fall. So, says Einstein, the Galilean-Newtonian concept of inertial systems is not good: such things do not exist in the universe. The real inertial system is obtained in a laboratory falling freely. From inside this cabin, you cannot even detect any curvature of space-time.

It should be pointed out that the 'inertial system' obtained in this way is only *locally* inertial. You cannot have a large laboratory which is entirely inertial. In other words, whereas an observer attached with a single particle falling freely cannot detect any curvature in space-time, *two such observers separated in space* can. Let us see how this is true and how it can be understood in terms of geometry.

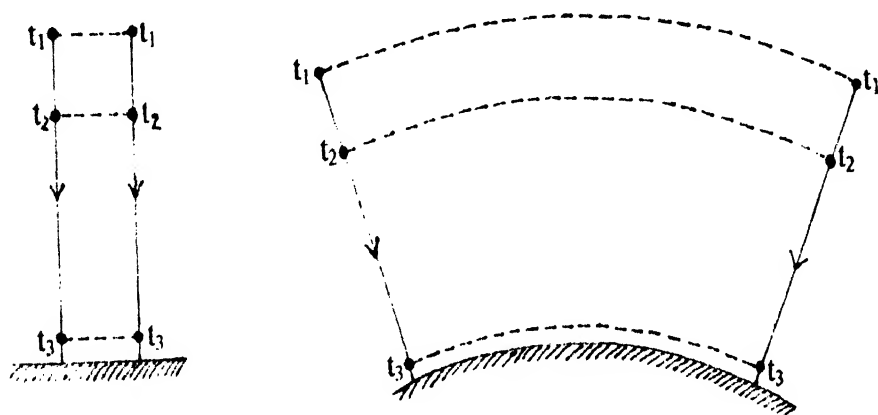


Fig.38: Detecting the curvature of three-dimensional space: (a) locally (not possible); (b) non-locally (possible)

In Fig. 38(a), we show two small iron balls being dropped from the same height above the Earth's surface. If the starting points of the two balls are not much separated, the paths of fall of both can be considered parallel. If there were an observer attached with each ball, each would find both balls as free particles and measure no changes occurring in their relative positions. However, if the starting positions of the balls are separated by a sufficient distance (comparable to the radius of the Earth), what will happen? As the balls fall towards the Earth's centre, their paths will not be parallel (Fig. 38b). Each of the observers will find the distance between the balls decreasing with time. It will be illustrated below that this decrease in distance may be taken as a sign of curvature of the space-time in which the balls are situated.

Curvature of a two-dimensional space is always easier to visualize than that of a four-dimensional space. Therefore, consider the Earth's surface with two travellers situated at points A and B on the equator, far separated from each other. Both start moving straight northward with equal speeds (Fig. 39). Since both are travelling 'in the same direction' (northward), they think their paths will be parallel. But, lo! They find the distance between them decreasing



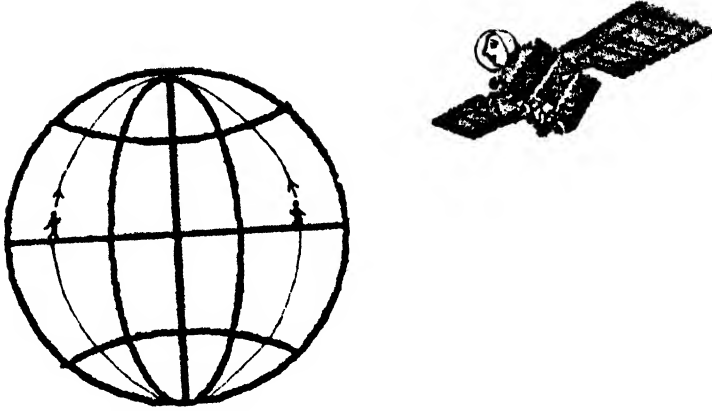


Fig.39: Detecting the curvature of the Earth's surface

with time and cannot understand why. Until an astronaut in a satellite overhead tells them the secret. Says he, "I see clearly from here that the surface on which you are moving is *curved*. Therefore, your separation goes on decreasing even though your paths are 'parallel lines'.

Indeed, in the case of two freely falling balls, too, it can be proved mathematically (with some trigonometry) that the decrease in separation obtained by considering the curvature of the space-time matches exactly the result obtained from the geometry of Fig. 38 (b).

The question may arise: is the concept of curved space just a new fashion, merely an attractive mathematical trick which makes us see well known gravitational effects as curvature of space? Or, does it actually give some results *different* from the good old Newton's Law of Gravitation?

The answer is that, indeed, Einstein's General Theory of Relativity gives some different results. Of course, as far as events of daily life are concerned, these are explained equally well by each theory. However, different results will be obtained upon consideration of very heavy bodies, very large distances and very long time-intervals. For example, Newton's Law says the force of gravitation of one body upon another varies in inverse proportion of the square of their distance ( $r^2$ ). According to Einstein's Theory, in the presence of a heavy body, the scales of space and time change, and the result is the same as if there were a force which varied *approximately* as the inverse square of the distance. More exactly, the apparent force has another small term which varies as the *inverse fourth power* of the distance ( $r^4$ ). Such a term is not present at all in Newton's Law.

In fact, the General Theory of Relativity has brought about an entirely new thinking about the nature of gravitation. In Newton's theory, two bodies can exert a force on each other even from a great distance. But Einstein's theory sees this effect in two stages. First, one heavy body causes a curvature

in the space-time around it. Second, as any other body moves in this curved space-time, its world-line does not remain straight but becomes a geodesic. Note that *this effect is purely local*, not action from a distance. Calculations based on curved space are a bit more complicated but yield more accurate results, as we shall discuss in the next section.

#### 4. Some Characteristic Results of the General Relativity

We have already seen that the characteristic results of the Special Theory of Relativity are not seen in daily life. For example, the percentage decrease in the length of an object moving with an ordinary velocity is so small that it cannot be measured with ordinary instruments. Likewise, it is not possible to distinguish the results of the General Theory of Relativity from those of Newton's Law of Gravitation under daily-life conditions. Making a distinction between the two would require extremely sensitive measurements of certain effects of the presence of very heavy objects. Several such measurements have been made from time to time, and the General Theory has been found to be correct each time.

One of these measurements is related to a precession (rotation) of the orbit of the planet Mercury. As you know, each planet revolves around the Sun in an elliptical orbit due to gravitational attraction. According to the laws of Newton and Kepler, any planet should go on moving *along the same orbit again and again*. However, detailed and fine measurements made over the last

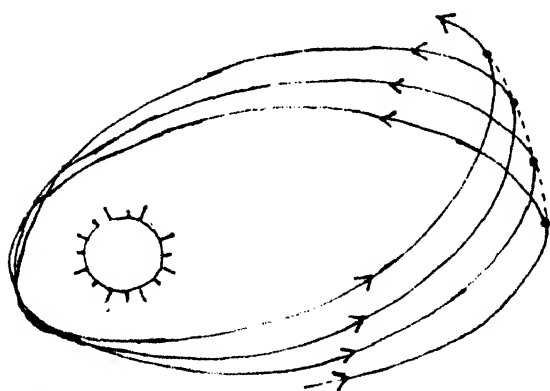


Fig.40: Precession of the orbit of Mercury

couple of centuries have revealed that it does not happen exactly that way. The perihelion (farthest point) of the orbit of a planet does not remain the same but moves slowly (Fig. 40). Scientists had already found one cause of this *precession*: the gravitational pull of other planets. When detailed calculations

were made on the basis of Newton's Law of Gravitation including the effects of other planets, the precession of planets' orbits could be largely explained. This was a further confirmation of Newton's Law.

However, super-fine measurements revealed a *small mismatch* with the calculated results mentioned above. In the case of Mercury, the perihelion of its orbit precesses by 9.55' every century\* whereas, according to Newton's

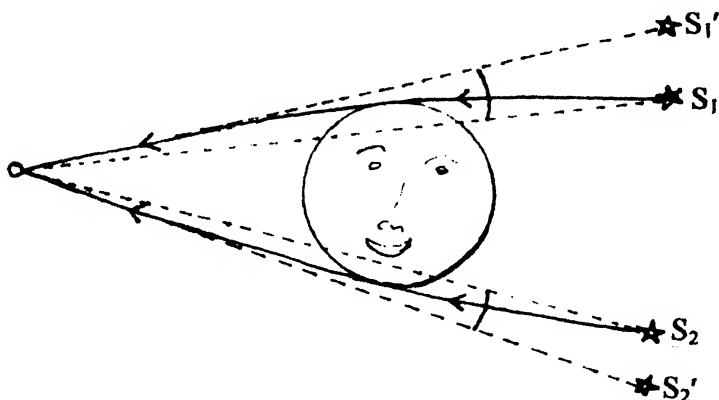
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\* One minute ( $1'$ ) means one-sixtieth of a degree ( $1^\circ$ ). One-sixtieth of a minute is called a second ( $1''$ ).

Law, this shift should be only  $8.85'$ . Therefore, there remains a difference of  $0.7'$  or  $42''$  per century. This discrepancy, though extremely small, is definitely present. Einstein showed that calculations made on the basis of a curvature of space give exactly the correct result for Mercury (and for all other planets).

The second measurement relates to the geometry of a light-ray. The curvature of space-time around a very heavy body affects not only the paths of material objects, but also of rays of light themselves. Before the General Theory, it was not obvious whether gravitation affects light or not, but now that we are talking about space-time itself getting curved, how can the world-line of light in it remain straight?

Einstein made a calculation on the basis of the General Theory, and showed that light-rays coming to us from a distant star will get deflected from their original path by an angle of  $1.75''$  if these come touching the periphery of the Sun on the way (Fig. 41). Even if light is considered as particles with mass,

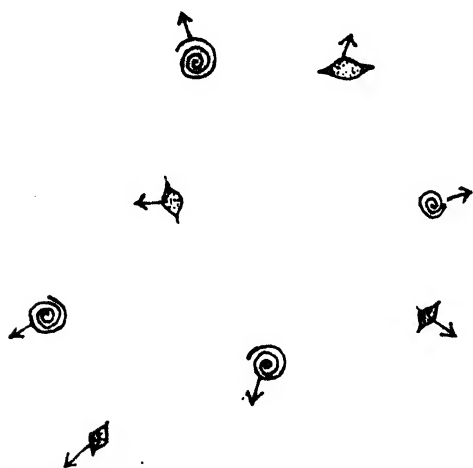


*Fig.41: Deflection of light rays in the vicinity of the Sun*

according to the equation  $E = mc^2$ , calculations based on Newton's Law of Gravitation give a deflection of only  $0.85''$ , i.e. one half of Einstein's value. Such a tiny angle may be considered negligible in most observations, but here it was crucial, and scientists had the instruments required to measure it accurately. The only problem was this: how to look at a star at all when the blazing Sun was present in the same general direction?

Einstein published his General Theory in 1916, and along with it the formula for the bending of light rays. When this news arrived in England, astronomers there decided to verify this effect at the time of the total solar eclipse in 1919. At eclipse time, light coming from the Sun would get almost cut off, and bright stars in its background would be seen. Thus, they planned observations at two places: Principe Island (Africa) and Sobral (Brazil). It was a time of war, in fact, war against Germany where Einstein was living. Yet, the scientific enthusiasm of the English astronomers carried them through all the preparations. The observations showed that the result of the General Theory was correct.

Even more definite confirmations were made in later decades using radio-astronomical techniques.



**Fig.42: The Galaxies running away from each other**

*shifted slightly towards the red end.* This 'red-shift' has been actually measured using ultra-sensitive instruments and this, too, has confirmed the General Theory of Relativity. Likewise, in huge gravitational fields, light has been found to travel in free space with a velocity somewhat smaller than its natural value  $c$ ; this, too, is in accordance with the said Theory.

The most exciting applications of the General Theory of Relativity have been in Cosmology. Among the new ideas developed in this area during the last few decades, perhaps the most dramatic is the possibility of *black holes*. This has been brought out clearly in the theoretical and experimental studies relating to the various stages of evolution of stars, as explained briefly below. A very heavy star, once it has used up all its nuclear fuel, begins to contract due to its own gravity. As its size becomes smaller, its atoms come closer together, and the fields of gravity become even stronger, thus pulling the atoms further in. If the original mass of the star is large enough, it can contract to such an extent that its density becomes hundreds of times the densities of atomic nuclei. Just try to imagine such an incredible density: it is as if the entire mass of the Sun were to be confined within a radius of 3 km!

According to the General Theory of Relativity, the space around such a super-heavy and super-dense body will be very far from flat. The curvature in the space around an ordinary star like the Sun is so small that it can barely be measured. But if a much larger mass is confined to such an unimaginably small size, would the curvature of space not take on dramatic proportions? Well, calculations tell us that, in such a case, the space will bend so much that it may close upon itself. The concept of one- or two-dimensional space 'closing upon itself' has already been explained in the last chapter (Fig. 27). In such a space, you cannot get out of a certain confined region, no matter how far you

The General Theory also predicts that all the clocks in a curved region of space-time will run a little slower. This implies that even the frequencies of electrons going around in various atoms will become less, and so also the frequencies of light emitted by them. Thus, the huge gravitational field on the surface of the Sun should cause the frequency of light emitted by atoms present there to decrease. In this way, all the colour-lines in the spectra of various elements in sun-light should be

travel, the same way that you can never get out of the Earth's surface by walking or sailing along it. Similarly, if four-dimensional space-time closes upon itself around a super-high-density star, *no matter will ever be able to come out of it*. Not even light, nor radio waves: after all, even they have to move in the same curved space-time. Such a star will, consequently, never be visible to you, but it will be present all the same. If you ever chance to pass near it in your space-ship, it will pull you into its interior by means of its intense gravity and you will be crushed into sub-nuclear matter and confined to the black hole for ever!

The General Theory of Relativity has found a crucial application in the field of Cosmogony. We already know that all the galaxies in this vast universe are running away from one another, and the universe is constantly expanding (Fig. 42). But how did this expansion start? And just how did our universe begin? Will this expanding universe go on expanding forever, or will the receding galaxies turn back and start approaching each other after some time? Of course, answering such questions involves comparing the effects of gravitational forces (which tend to pull the galaxies together) and of relative motions (which tend to continue the expansion). Newton's Law of Gravitation, which may be taken as a close approximation to reality in planetary motions, etc., is too gross when considering the huge distances and masses involved here. The equations necessary for an accurate analysis, therefore, must be based on the General Theory of Relativity. That is why the Theory is still at the centre of the latest researches in this field, over eighty years after its formulation.



### *Where Is the Need for an Observatory?*

*Einstein was on a tour of California (U.S.A.) with his wife Elsa. They were taken to see the Mt. Wilson Observatory with the huge telescope of 200 inch diameter. Mrs. Einstein asked about the purpose of such a large telescope and the associated paraphernalia. She was told it was used to study the structure of the Universe. Elsa, who was a simple housewife, said innocently, "Is that so? My husband can do this work at the back of a used envelope!"*

### *A Friend in Need*

Einstein was always ready to help anybody in need. At Princeton, one of his scientific assistants, Leopold Infeld, was close to the end of his one-year contract, and the authorities had declined to give him an extension. Infeld (a Jew) had found a shelter at Princeton after escaping from Hitler's clutches; where could he go now? Anyway, he had a strong desire to work with Einstein for some more time. He hit upon a bright idea: why not to write a book with Einstein? With that great name on the book, any publisher would gladly take it up, he thought. Even if the publisher paid half the amount of the royalties in advance, he (Infeld) would be able to live on it for a year. So, he summoned all his courage and put the proposal for jointly writing a book before Einstein. As the latter began to think, Infeld added, "This will enable me to stay here and work with you for some more time, which I need." Hearing this, Einstein said, "That is not a bad idea at all", and shook hands warmly with Infeld. In time, this book was published under the title *The Evolution of Physics*.

Then there was an artist, an admirer of Einstein, who wanted to paint his portrait. He consulted one of Einstein's friends as to how to approach him, "What?", said the friend. "Do you think Einstein will have the time to sit for so long just for the portrait? Impossible!" He knew that Einstein cared neither for fame nor for portraits. Nevertheless, the artist did find a way. He said to Einstein, "I am going through an acute financial crisis. If only I could paint your portrait, the market value of my paintings would go up." Thereupon, Einstein gladly agreed to sit and pose for the portrait.

## Seven *Paradoxes in Relativity*

*Some mind-boggling paradoxes based on relativistic concepts of simultaneity, time-dilation, length-contraction, inertial reference-frames and the Principle of Equivalence are presented, together with solutions.*

### **1. Common Sense is Baffled!**

The Theory of Relativity has shaken many of our common-sense notions developed through experiences of day-to-day life. It has become clear that ordinary common sense is not enough in analyzing and solving problems in which velocities close to the velocity of light  $c$  are present. Often we get baffled in the process of finding answers to such questions. Sometimes, the same question appears to yield two different, mutually contradictory, answers. If you look at the problem from one angle, you get one answer, and if you look from another angle, a different and contradictory answer is obtained.

It is not as if such a puzzling situation has arisen for the first time in human history. Whenever a new, revolutionary discovery is made which expands the frontiers of our knowledge, old definitions undergo changes. Our common sense, fed upon old notions that worked well within their limited domains, cannot readily digest the new concepts that apparently contradict the old ones. Let us consider the same old example – roundness or flatness of the Earth. So many confusions and puzzles have arisen regarding this. If the Earth is round, then do people living on its opposite side stand on their heads? Won't those standing on the sides fall down into the recesses of space? This was a great puzzle at one time. We have already seen that its solution is that the word 'down' does not indicate the same direction for people all over the world. Of course, if we confine our attention to a small region, where all people are within a few kilometres of each other, then 'down' has nearly the same meaning for all, but when two observers are separated by thousands of kilometres, it becomes necessary to understand its meaning more fully. In the same way, we cannot depend on our old common sense to correctly and fully understand measurements made with reference to laboratories moving at very high speeds or events occurring in the vicinity of very heavy or very

dense bodies. In this chapter, we shall present a few illustrative situations and puzzles. The puzzles given here are rather simple; other, more complicated ones, can be found in formal books on Relativity (like "Space-Time Physics" by Taylor). Solutions of the paradoxes presented here are given at the end of this chapter.

## 2. The Twin-Paradox

This well-known paradox is based on time-dilation. We have seen in Chapter Three that if the time-difference between two events occurring at the same point in a particular laboratory is  $T_0$  as measured in that laboratory, then this time-interval will be measured as  $T' = T_0 / \sqrt{1-v^2/c^2}$  in another laboratory moving at velocity  $v$  relative to the first one. Of course,  $T'$  is larger than  $T_0$ . Now, suppose there are two identical twins, Ramesh and Suresh. Ramesh goes on a long journey to distant stars on a fast-moving space-ship, leaving

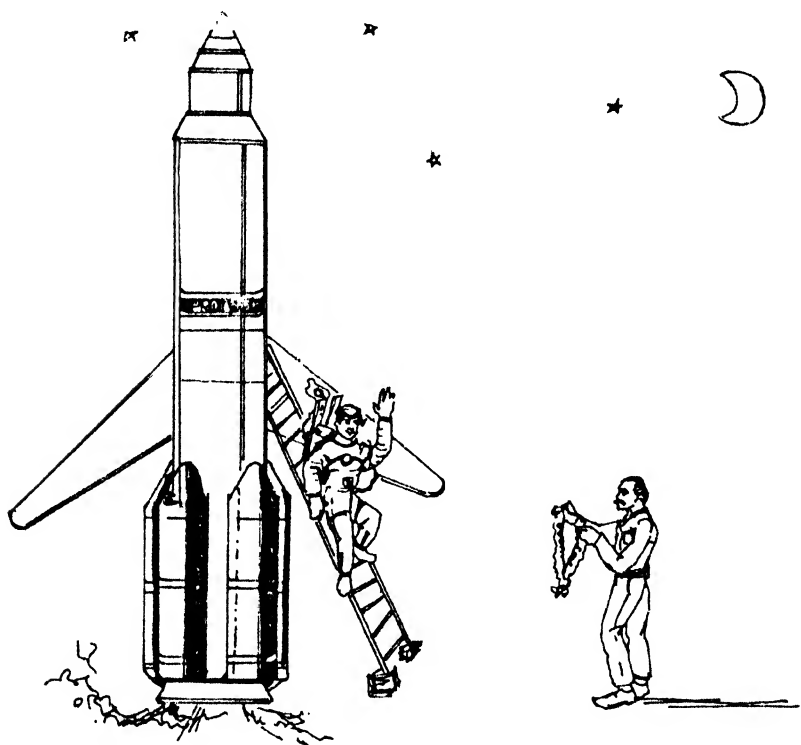


Fig.43: Of the two twins, who will be older?

Suresh behind to take care of the fields (Fig. 43). If Ramesh's space-ship has a speed which is 90% of the speed of light, the multiplier of time will be  $1/\sqrt{1-0.9^2} = 2.3$ . This means that if Ramesh, sitting in his space-ship, takes 10 minutes in sipping tea, the Earth-bound Suresh will see this tea-session as lasting  $10 \times 2.3 = 23$  minutes.

This time-dilation applies not only to tea-drinking, but to every single event



or change taking place inside the flying space-ship. Suppose Mr. Ramesh wanders about in space for twenty years (according to his own clock and calendar), and then returns home to the Earth. Naturally, his age will have increased by twenty years. However, on the Earth,  $20 \times 2.3 = 46$  years will have passed by then, and his twin brother Suresh will have become an old retired man.

But wait a minute. While the space-ship is in motion relative to the Earth, the Earth too moves relative to it. Thus, the time-dilation discussed above can be applied the other way too. That is, while Ramesh spends twenty years wandering in space, does he not see the Earth-clock ticking more slowly than his own, recording only  $20/2.3 = 8.8$  years? "So what?", you might say nonchalantly. "May be both are right." But when the two brothers actually meet, and have an opportunity to greet each other, *who will be the older of the two?* And who will touch whose feet? Or will both be of the same age ....?

### 3. The Bamboo and the Barn

Let us have a riddle regarding lengths too. There is a barn (Fig. 44), 4 metres long. There is a bamboo whose length too is 4 metres. Of course, when we say something is so many metres long, it is understood that it is the *rest-length*, i.e. its length as measured by an observer who is stationary relative to it. So, the doors of the barn can be just barely closed, keeping the bamboo

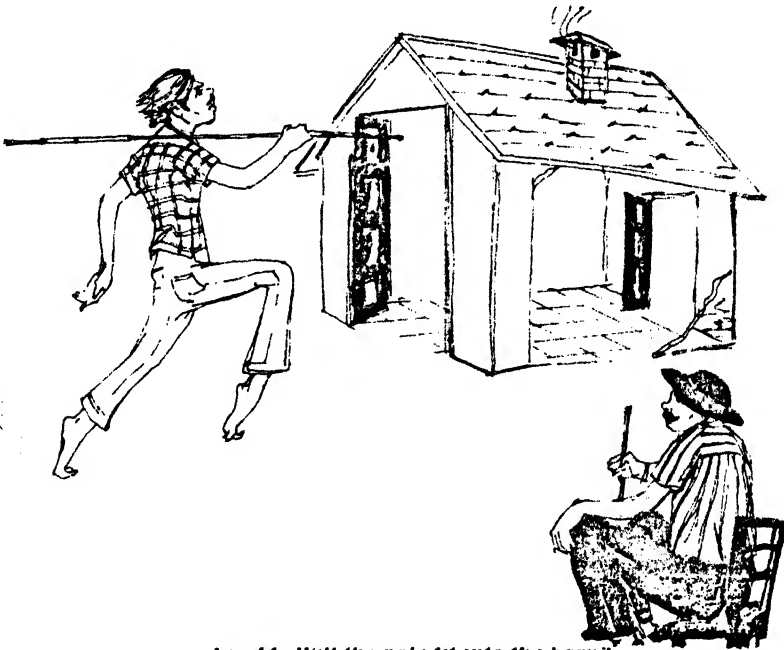


Fig.44: Will the pole fit into the barn?

on its floor along one wall.

Now, if a runner picks up the bamboo and runs at a very high speed

parallel to its length, enters one door of the barn, crosses its length, and runs out of the opposite door, *will the bamboo be completely inside the barn at any moment?* The attendant sitting outside the barn will see the length of the bamboo contracted in accordance with the relativistic effect; therefore, he will answer “yes”. But the runner sees the barn as moving and the bamboo as stationary. Certainly, he will see the length of the *barn* being less than that of the bamboo, and his answer will be “no”.

So, dear reader, what is your opinion: will the bamboo fit into the barn or not?

#### 4. Space-war

Imagine one rocket each belonging to U.S.A. and Russia, both being of the same make, model and company. Therefore, their lengths are exactly equal when lying in the store-house. Now, both of them, flying in opposite directions in space, whiz past each other (Fig. 45). The Russian rocket is fitted with a gun at its back end. As soon as the front end of this

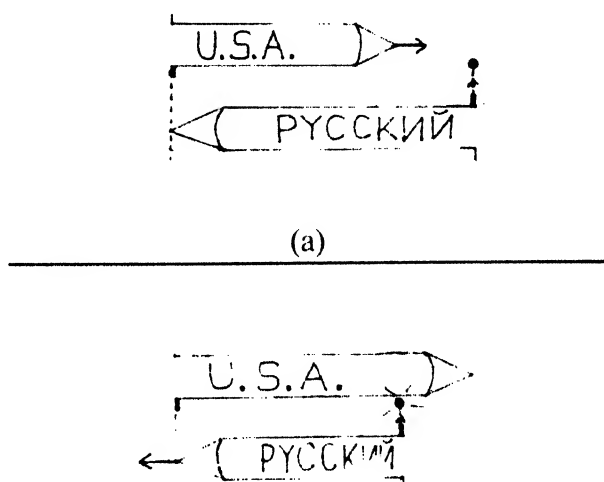


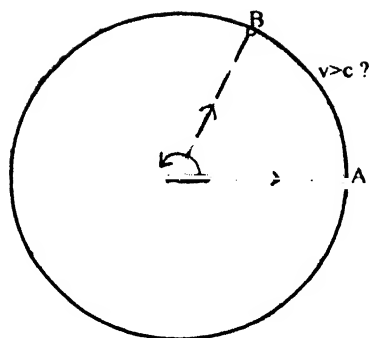
Fig.45: Will the Russian shell hit the American space-ship? (a) No (Russian observer's viewpoint); (b) Yes (American observer's viewpoint)

rocket reaches beside the back end of the American one, the Russian rocket fires a shell from its gun perpendicularly towards the American rocket. Fig. 45(a) shows the situation as seen by an observer moving along with the Russian rocket. According to him, the Russian rocket is stationary, but the American one is moving and hence is contracted in length. That means the shell will not hit the American rocket. On the other hand, the view of an observer moving along the American rocket is that it is the Russian rocket which is moving, and whose length is shortened (Fig. 45 b). Therefore, when the gun fires the shot, part of the American rocket is still opposite to the gun and it will be hit.

What will happen actually: *will the American rocket get hit or not?*

#### 5. Faster than Light?

Fig. 46 shows a laser giving out an intense beam of light. As you will be aware, a laser can send out a narrow beam of light to very large distances.



**Fig.46: The laser beam and the spot on the wall**

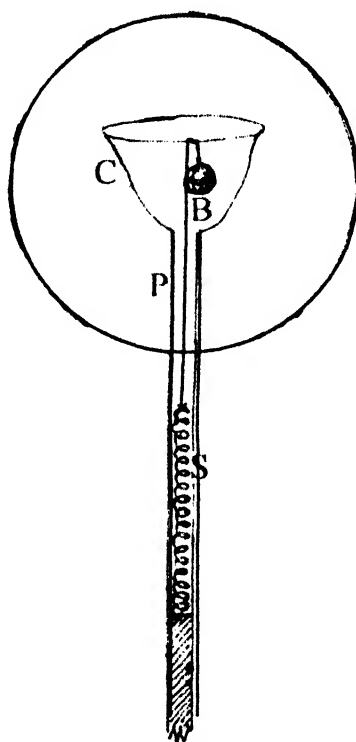
Now imagine a circular wall such that the laser is at its centre. If the laser is rotated rapidly on a vertical axis with the help of a motor, the light spot on the wall will also rotate, touching various points (like A and B) one after the other.

Now, if the radius of the circular wall is increased, then the speed of the light spot rotating along the wall will also increase in the same proportion. Clearly, there is no limit to the increasing radius of the circle. So, with a sufficiently large radius of the circle, will the light spot rotating on the wall not travel faster

than  $c$ , the speed of light? (For example, if the laser makes 1000 rotations per minute, which is not impossibly high, we need to increase the radius of the wall to just about 3000 km for the light spot to travel on the wall faster than  $c$ .) And will this not contradict the theory of relativity?

## 6. Einstein's Puzzle-Toy

When Einstein was living in Princeton (U.S.A.), his neighbours Prof. and Mrs. Rogers often presented him with small toys or puzzles. On Einstein's seventy-sixth birthday, they made for him a wonderful little toy incorporating a challenging puzzle. This toy is shown in Fig. 47. It has a transparent little globe, at the centre of which is a transparent cup. There is a hole at the bottom of the cup from which a long transparent pipe is attached. In the pipe there is a spring whose lower end is fixed to the bottom of the pipe. The upper end of the spring is tied to a string which goes up the pipe and through the cup to outside the cup where a small brass ball is tied to the other end of the string. To begin with, the brass ball is hanging outside the cup and, due to its weight, the string is in tension and the spring is stretched. Now, the spring does not exert enough force to pull the ball into the cup, but if it were a little stronger, the ball would be pulled in and be situated on the hole in the cup.



**Fig.47: How can the ball be brought into the cup?**

The whole apparatus is mounted on a four-foot long bamboo. Now, the problem is *how to bring the brass ball into the cup*. By moving the apparatus at random, perhaps this might be accomplished by chance. But one has to find a definite and sure method.

## 7. Here Are the Answers

In the Twin Paradox, we must not forget that all results of the Special Theory of Relativity have been obtained on the condition that the laboratories concerned are inertial, i.e. moving at uniform velocities, with no change occurring in speed or direction. Now, the laboratory of Suresh, which is stationary on the Earth, satisfies this condition to a reasonable extent. But Ramesh's laboratory attached to his space-ship does not, because first it *starts increasing its speed* from rest, then goes to far-off distances, then *stops* and *turns back* and *accelerates* again, then *brakes* on arrival near the Earth and *stops* (Fig. 48). Even if we say that Ramesh does not start from a stationary state on the Earth but whizzes past Suresh, and does not stop on meeting Suresh again, he must turn back in the middle. Thus, the measurements made

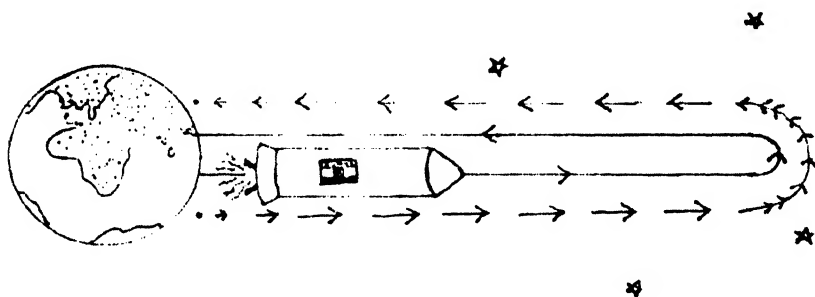


Fig. 48: Ramesh's lab (space-ship) is not inertial.

from his point of view cannot be treated by equations of the Special Theory. Only Suresh's measurements are valid. He sees that, at any given moment, Ramesh is moving with a certain speed and his (Ramesh's) clock is running correspondingly slower. Combining these various instantaneous observations, it is all right to conclude that 20 years of Ramesh's life overlap with 46 years of Suresh's life and, when the two brothers meet again, it is Suresh who will be older.

You might wonder, it is all right as an exercise in thinking, but is there a direct experimental proof of this? Well, in 1971, Hafele and Keating took a few very precise atomic clocks, synchronized them (matched their times) in Washington, D.C., and flew these around the globe, some clocks eastward and some westward, aboard fast jet-planes. Upon their return, their times were compared with 'stationary' clocks. It turned out that the *westward flying clocks gained 273 ns* (ns or nanosecond is a billionth part of a second) and the *eastward-flying clocks lost 59 ns*. This was taken to be consistent with what we have said above. Of course, you must remember that the clocks

which stay back in Washington, D.C., are not to be considered inertial since the entire Washington was rotating eastwards along with the Earth. Thus, the westward-flying clocks were more inertial. The east-flying clocks were the most non-inertial. Hence, we should expect these east-flying clocks (like the space-flying Ramesh) to lose time, which is what was actually seen. Detailed calculations show that the results tally exactly with the expected values.

In the second problem the question is: will the bamboo fit in the barn or not? There is no doubt that the bamboo will pass through the barn; what the question really amounts to is this: will the two ends of the bamboo be within the barn *at the same time*? It must be remembered here that the meaning of 'simultaneity' is not the same for both the barn-keeper and the runner. (The two ends of the bamboo are not at the same point in space.) Therefore, the keeper sees the two ends of the bamboo inside the barn at the same time, whereas the runner sees the back end of the bamboo entering the barn only after the front end is out. Both are equally right!

As far as the space-war-question is concerned, a careful reading reveals that it too involves the concept of *simultaneity* which is hidden in non-technical words. When is the gun fired? "At the moment when the front end of the Russian rocket is beside the back end of the American rocket." So the following two events are described as occurring at the same time: (i) the Russian rocket's front end and the other one's back end being next to each other, and (ii) firing from the back end of the Russian rocket. Note that *the two events are not occurring at the same place*; therefore, both observers will not agree about the events being simultaneous. The conclusion is that two different results will be obtained under different circumstances. If the two events occur simultaneously *according to observations made from the Russian rocket* (Fig. 45a), then the American rocket will not be hit. In that case, the American observer will *not* see the two events occurring simultaneously. What he will see is that the gun is fired *before* the front end of the Russian rocket reaches the back end of the other one, in fact, before the gun came beside any part of the American rocket. But if the two events occur simultaneously *according to the American observer*, as in Fig. 45 (b), then the American rocket will get hit. According to the Russian observer, this will happen because the gun will be fired *after* the nose of the Russian rocket has crossed the American rocket.

The next paradox relates to the rotating laser beam (Fig. 46). Yes, it is possible for the light spot to 'move' from point A to B faster than light. But so what? Actually, *no material or signal is moving from A to B*. The laser beam is not carrying any message along the wall from A to B; it can carry a message *from the centre* to A, B and all other points. Therefore, there is no violation of the postulate of relativity that no object or signal can move faster than  $c$ .

The last problem about Einstein's toy may appear to you to be not related

to the Theory of Relativity at all! There is no fast-moving space-ship in it, nor does it involve a super-heavy star. However, when Einstein was presented with the challenge to 'cup the ball' (Fig. 47), he immediately recognized that the answer lay in his 'Principle of Equivalence'. He lifted the apparatus high up and released it to fall straight down. The ball immediately came down into the cup. How? Simple! According to the above principle, the freely falling ball becomes weightless, but the spring continues to pull on it. As a result, it gets pulled into the cup.



***"Yes, I Will Recognize You"***

*At Princeton, Einstein once went to see the movie the Life of Emile Zola with his fellow scientist Leopold Infeld. They gave their tickets at the door and entered the waiting hall. It turned out that they would have to wait for 15-20 minutes. Einstein felt like strolling in the open air outside. As they went out, Infeld told the door-man, "We shall be back in a few minutes." But Einstein was somewhat unsure, and very innocently asked the door-man, "We have given our tickets to you. Will you recognize us?" The door-man smiled and said assuringly, "Yes, Professor Einstein, I will recognize you." Einstein was always oblivious of the fact that the world adored him, and that in a small town like Princeton, every child knew him.*

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## About the Book

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